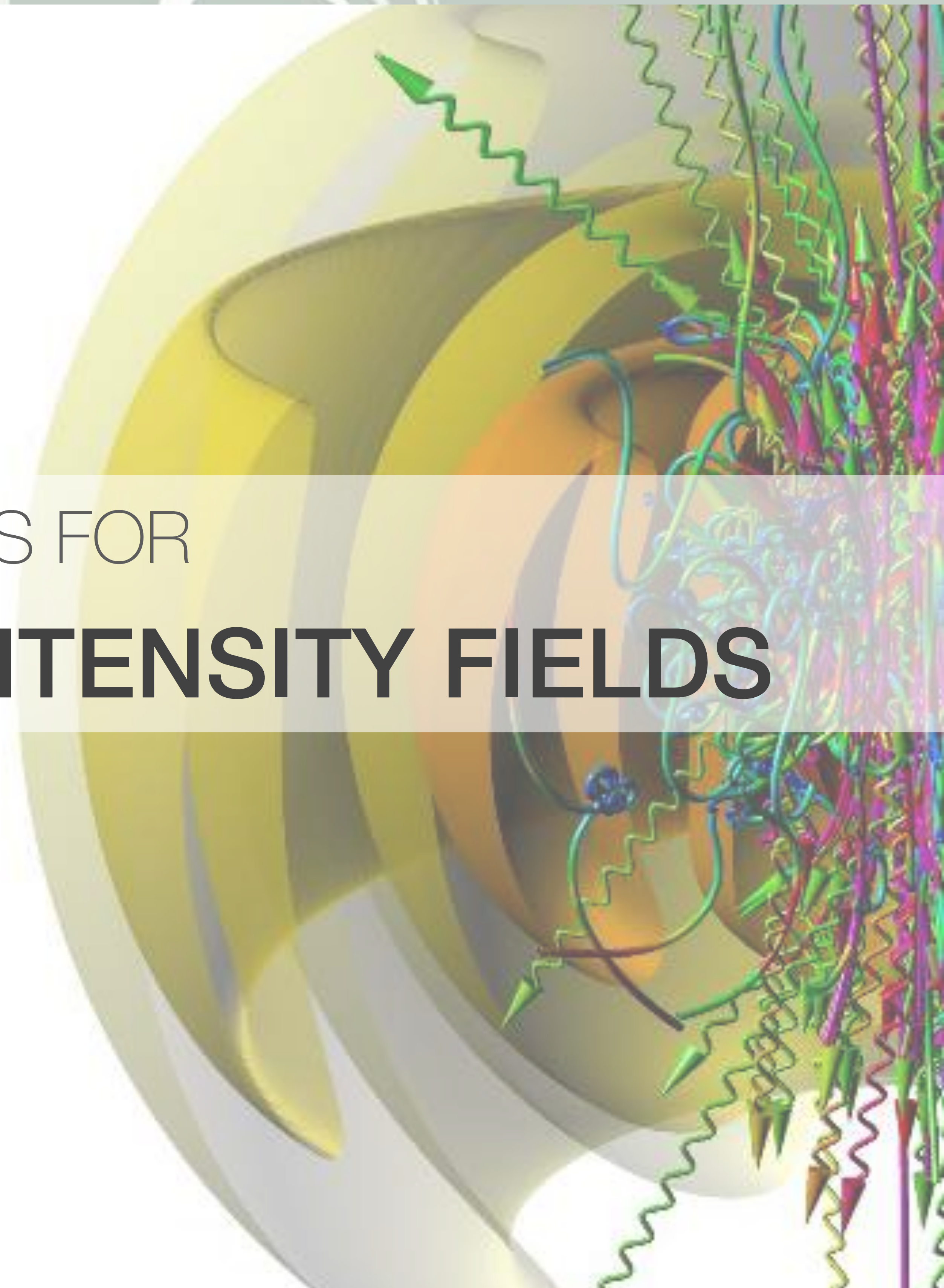
REVIEW OF NUMERICAL METHODS FOR
THE STUDY OF HIGH-INTENSITY FIELDS



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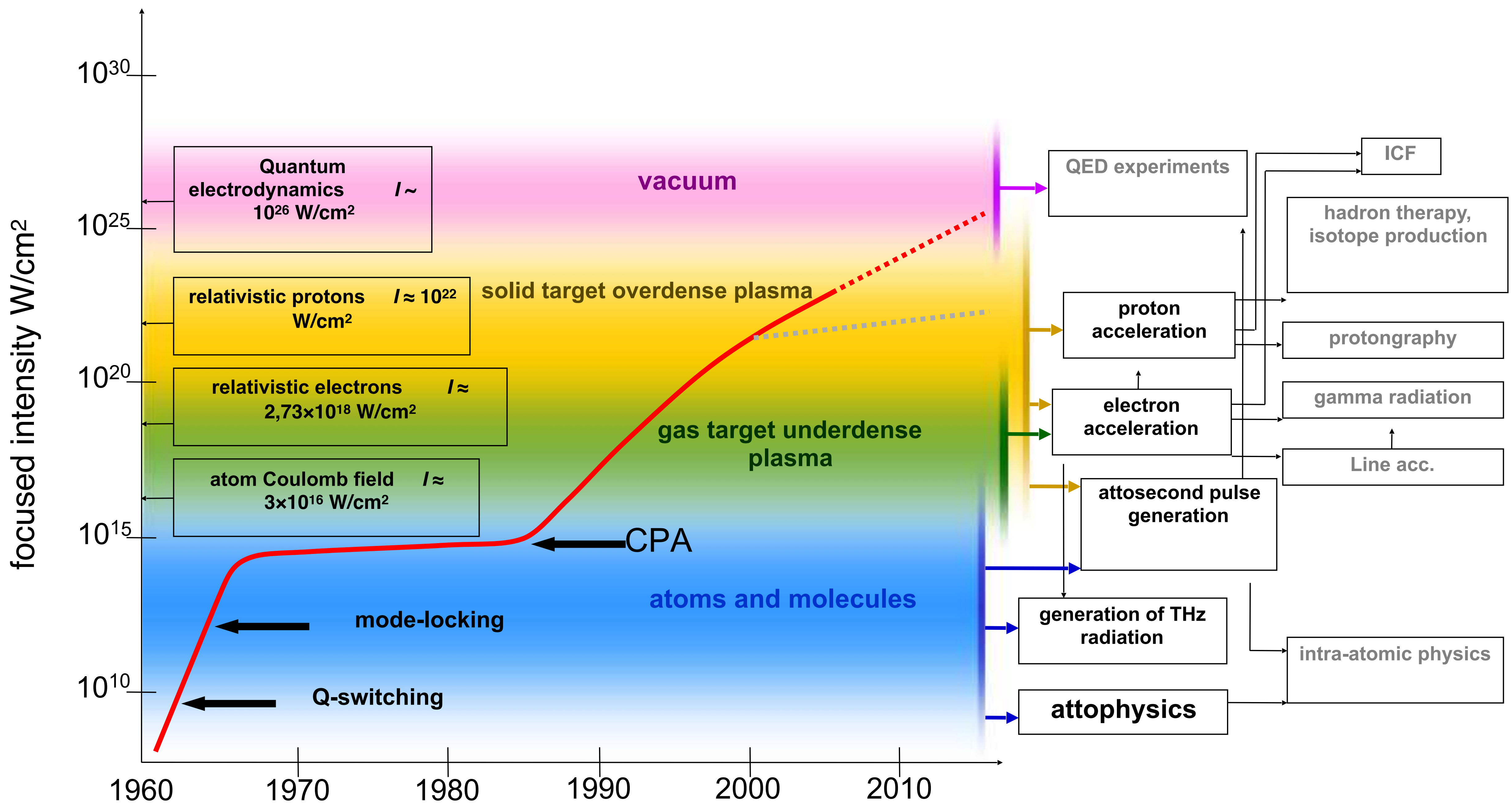
Outline

- Why we are here: strong-field physics using lasers is a growing field of research.
- A large number of laser facilities makes new experimental regimes available.
- Need numerical tools for many strong-field applications.
- However, numerical tools are built on analytical results and experimental input.
  - The numerical tools we develop are never better than the analytical input.
  - Out understanding of the outcome of complex numerical efforts relies on our analytical interpretations.
  - Benchmarking numerical results.
- Will therefore focus on the analytical basis for the numerical implementations.
- Open questions for future research.
- Literature (non-exhaustive):

— Marklund & Shukla, RMP 78, 591 (2006)
— Di Piazza et al., RMP 84, 1177 (2012)
— Ridgers et al., J. Comp. Phys. 260, 273 (2014)
— Gonoskov et al., Phys. Rev. E **92**, 023305 (2015)
— Arber et al., Plasma Phys. Control. Fusion 57, 113001 (2015)
— Vranic et al., Comp. Phys. Comm. 204, 141 (2016)

Laboratory plasmas in strong fields



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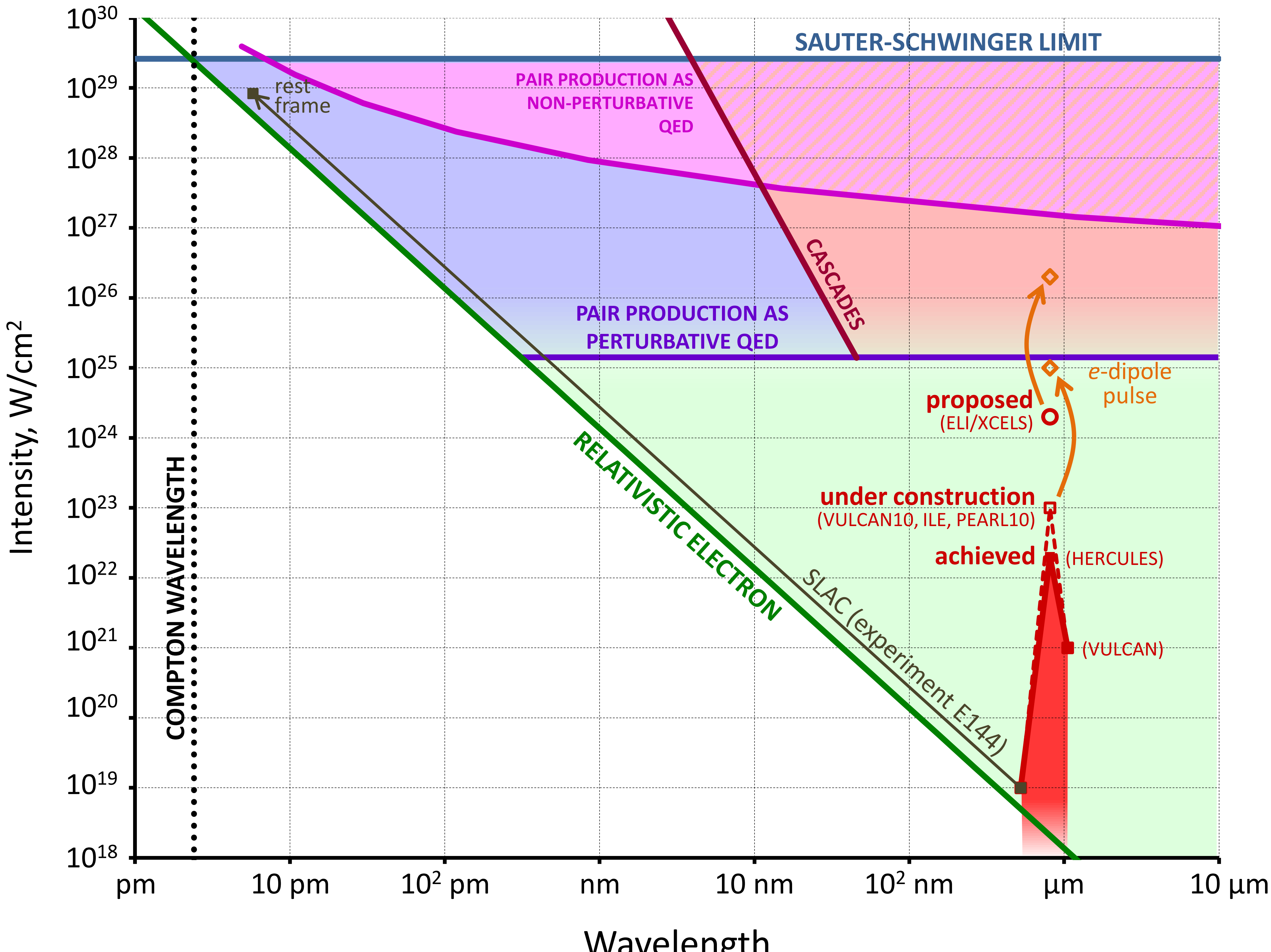
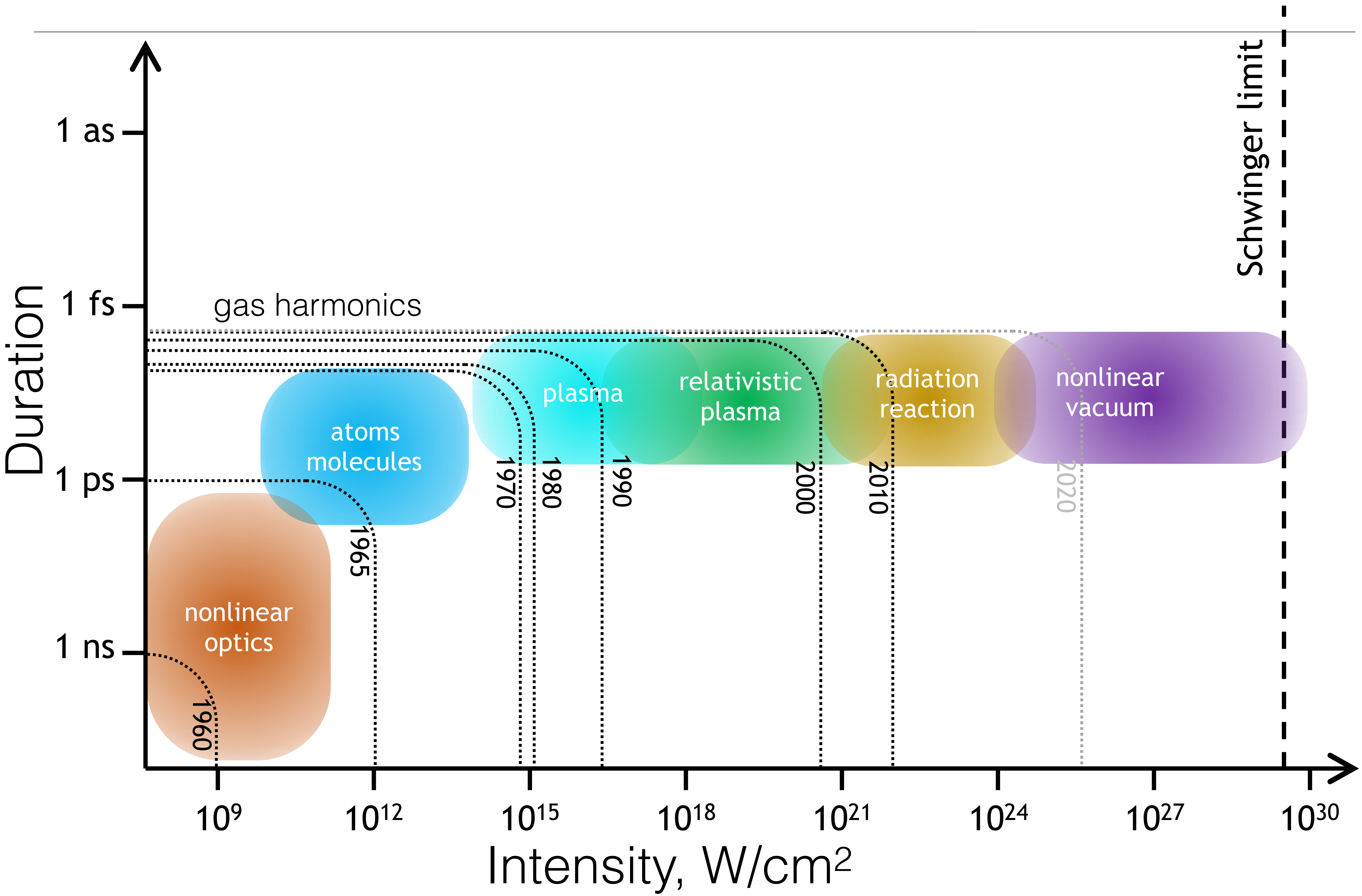
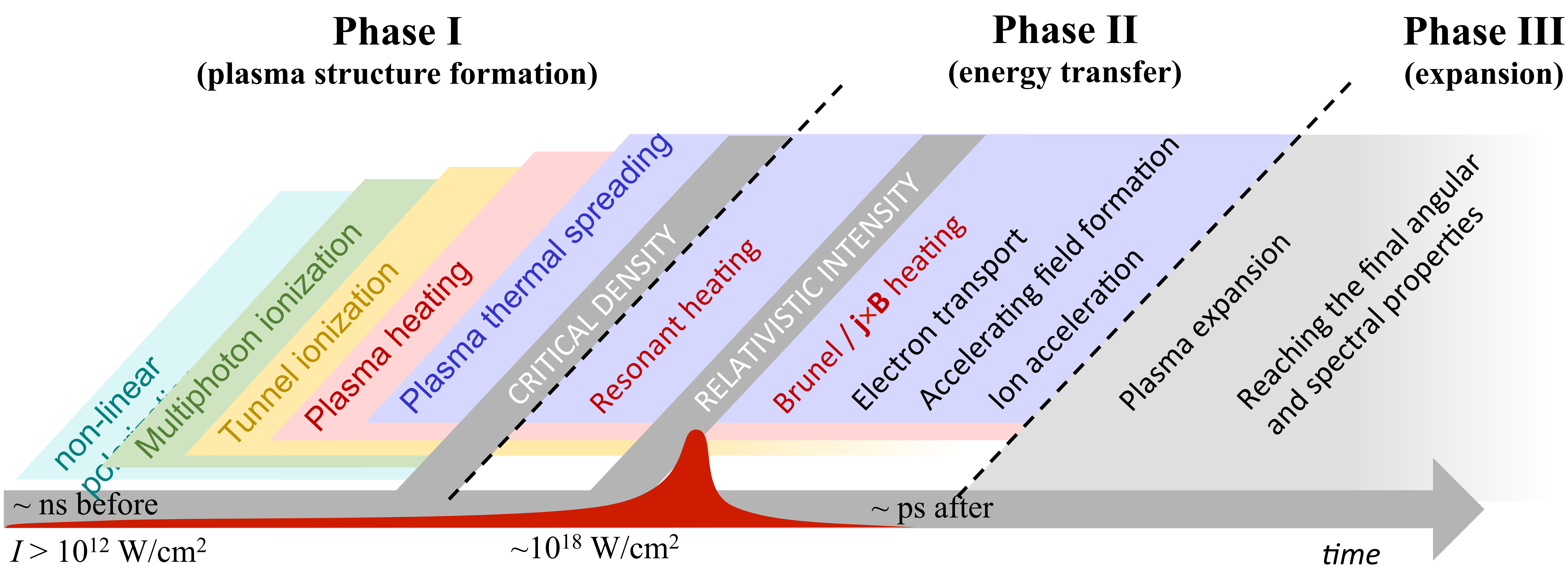


Evolution of intensity and duration



Processes in plasma formation



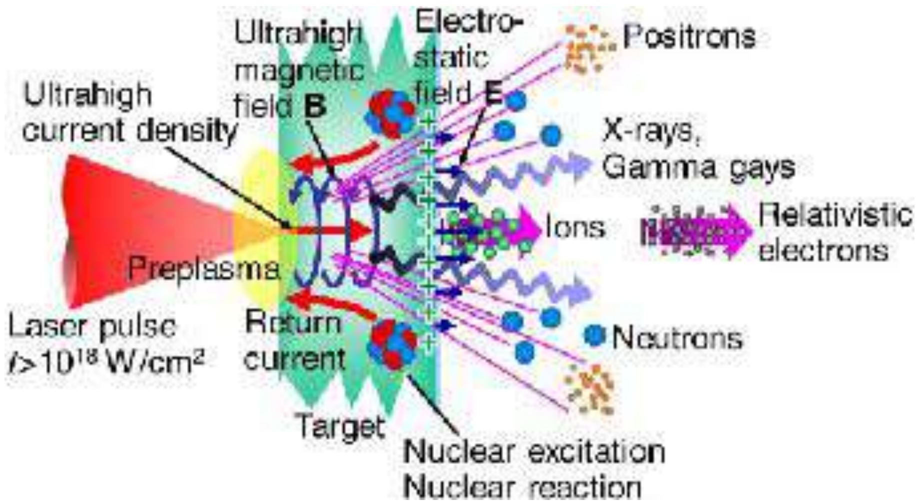
Atoms: TDSE, ionization theoretical/empirical rates

Plasma: Vlasov equation, MHD, other kinetic approaches

Plasma: Particle-In-Cell
- Maxwell's eq. (FFT, FDTD+)
- Particles motion (relativistic solvers)

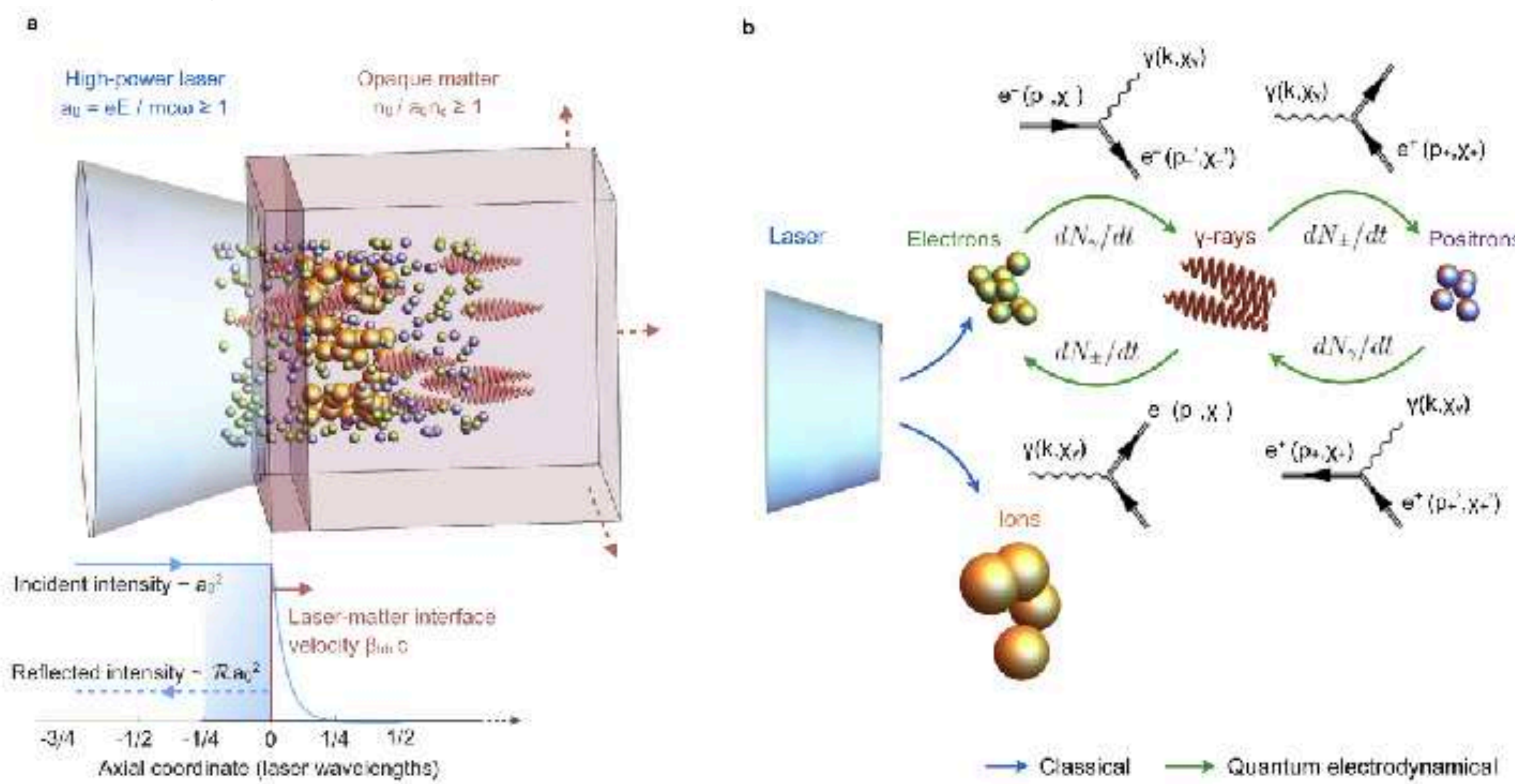
Particles:
- Maxwell's eq. (FFT, FDTD+)
- Particles (ballistic propagation)

Plasmas in strong fields



Daido et al., Rep. Prog. Phys. **75**, 056401 (2012)

How do we capture all of the physics, contained in such interactions, via a computational scheme?



Levy et al., arXiv:1609.00389 (2016)

Micro- and macroscopic physics

Particle simulation of plasmas

John M. Dawson

Department of Physics, University of California, Los Angeles, California 90024

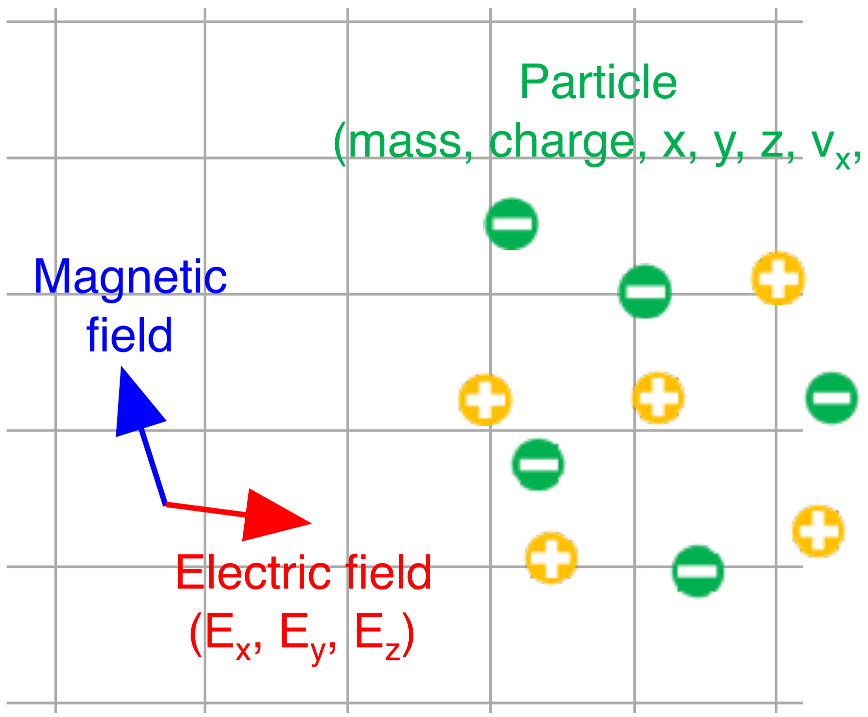
Reviews of Modern Physics, Vol. 55, No. 2, April 1983

”Proper treatment of systems where both the microscopic and macroscopic behavior are important will undoubtedly challenge simulation physicists for many years to come”

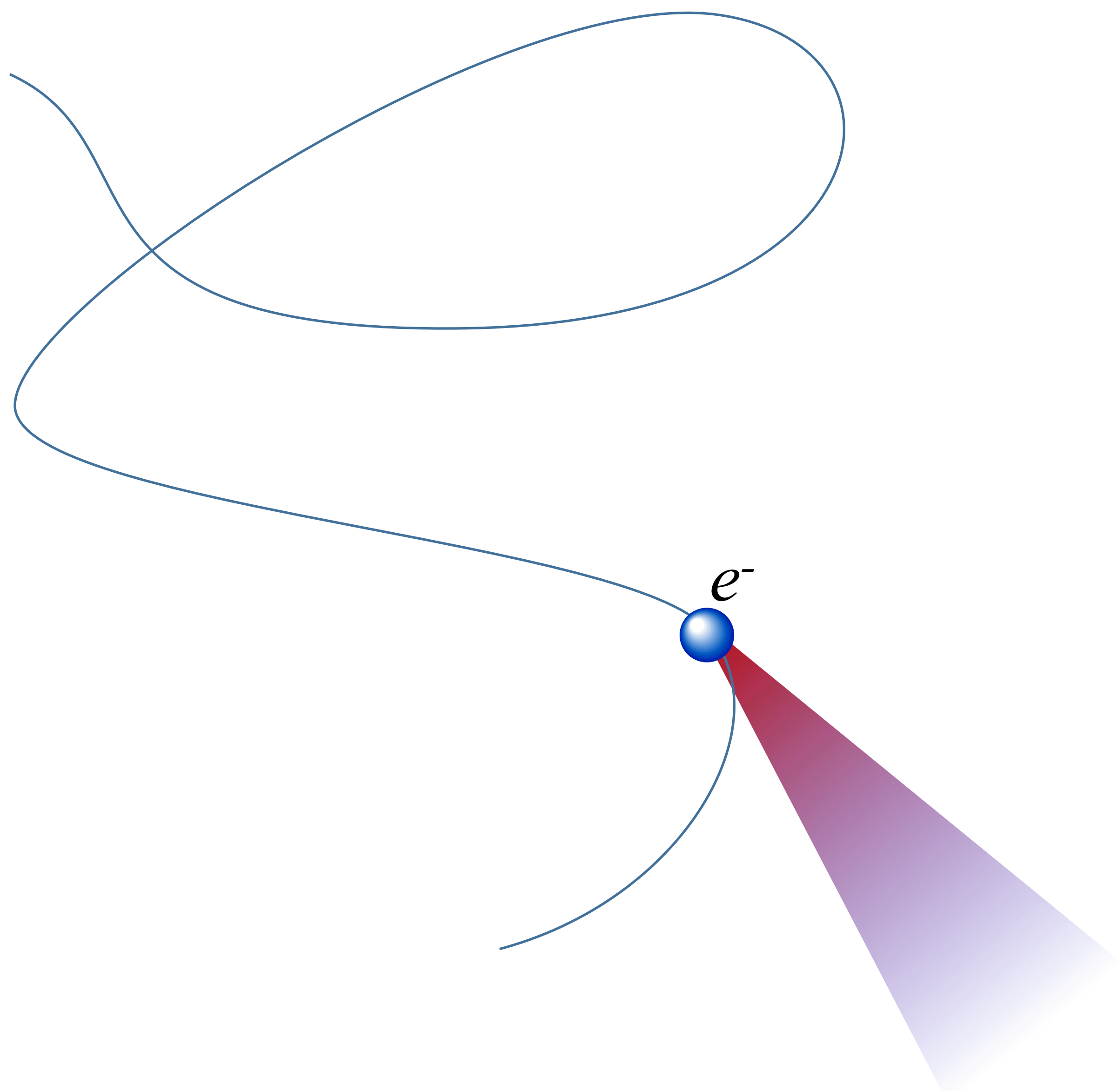
- Not only multi-scale, but also different treatment of physical quantities (e.g. EM fields vs. photons), collective effect and discrete events etc.

Plasma descriptions

- **Single-particle dynamics**: no backreaction, only particle motion in external fields.
- **Kinetic descriptions**: distribution function in phase space describing ensemble of particles, either in external fields or self-consistent.
- **Fluid models**: moments of the distribution function with closure assumptions.
- **Hybrid schemes**: treating plasma as a mixture of kinetic and fluid components.
- For some applications, such as ICF, **strong coupling** effects need to be accounted for (DFT and TDDFT).
- **Particle-in-cell** schemes belong to the kinetic category. Impose a grid structure.



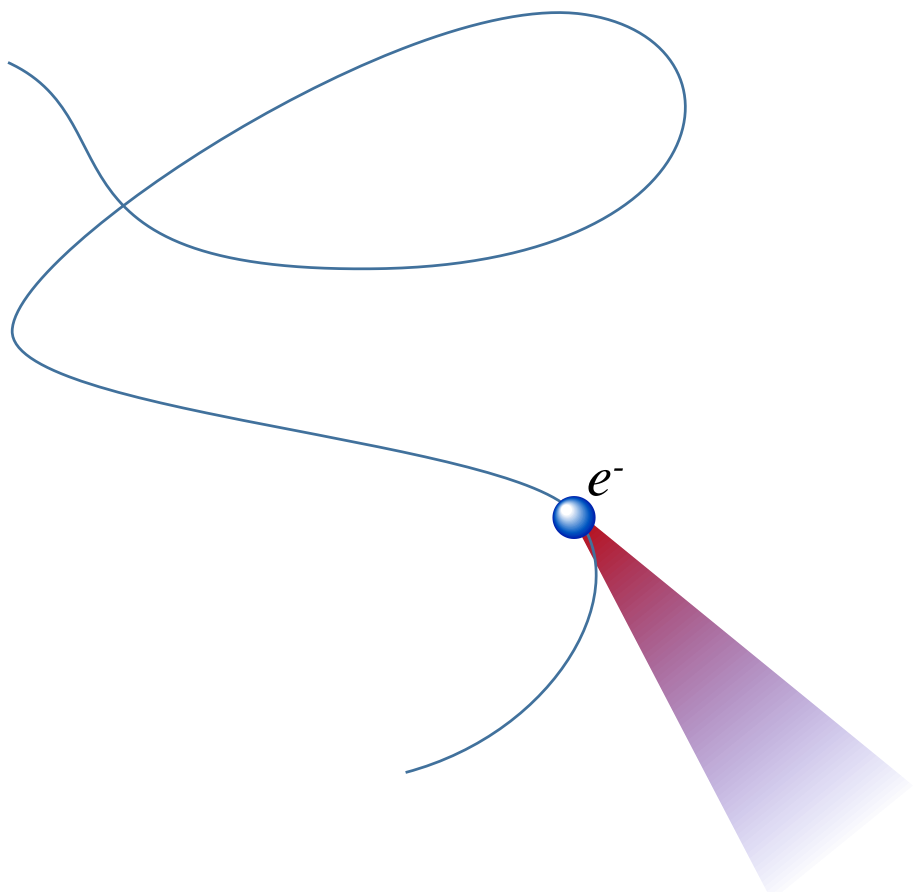
Numerical challenges



- take into account classical RR
- avoid double-counting by LF and RR
- chose model for classical RR
- take into account quantum RR
- control infra-red cutoff
- combine classical and quantum RR
- avoid double-counting by classical and quantum RR
- take into account pair production
- resolve time-scales
- control computational demands
- account for EM-field depletion
- account for multi-photon processes
- …

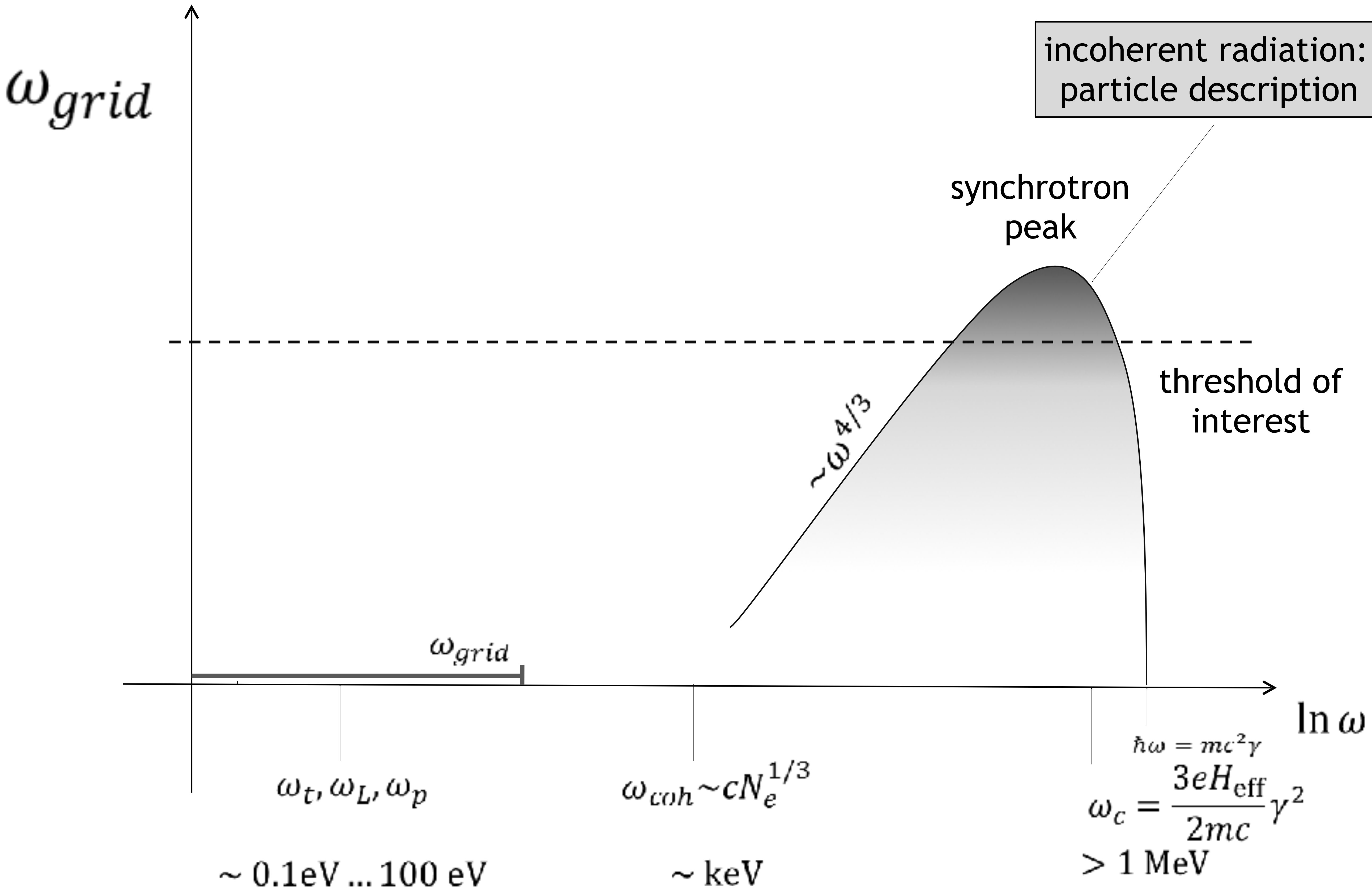
Synchrotron emission and radiation reaction

- In the strong acceleration regime, electrons emit large number of high energy photons (in high-*Z* targets, bremsstrahlung is a major contributor for moderate intensities).
- Typical frequency: $\omega_c = \dfrac{3eH_{\text{eff}}}{2mc}\gamma^2$,
- Making inclusion of synchrotron emission via direct solving of Maxwell's equations impossible in PIC scheme. *Cannot achieve good enough grid resolution.*
- Luckily the power spectra helps us.



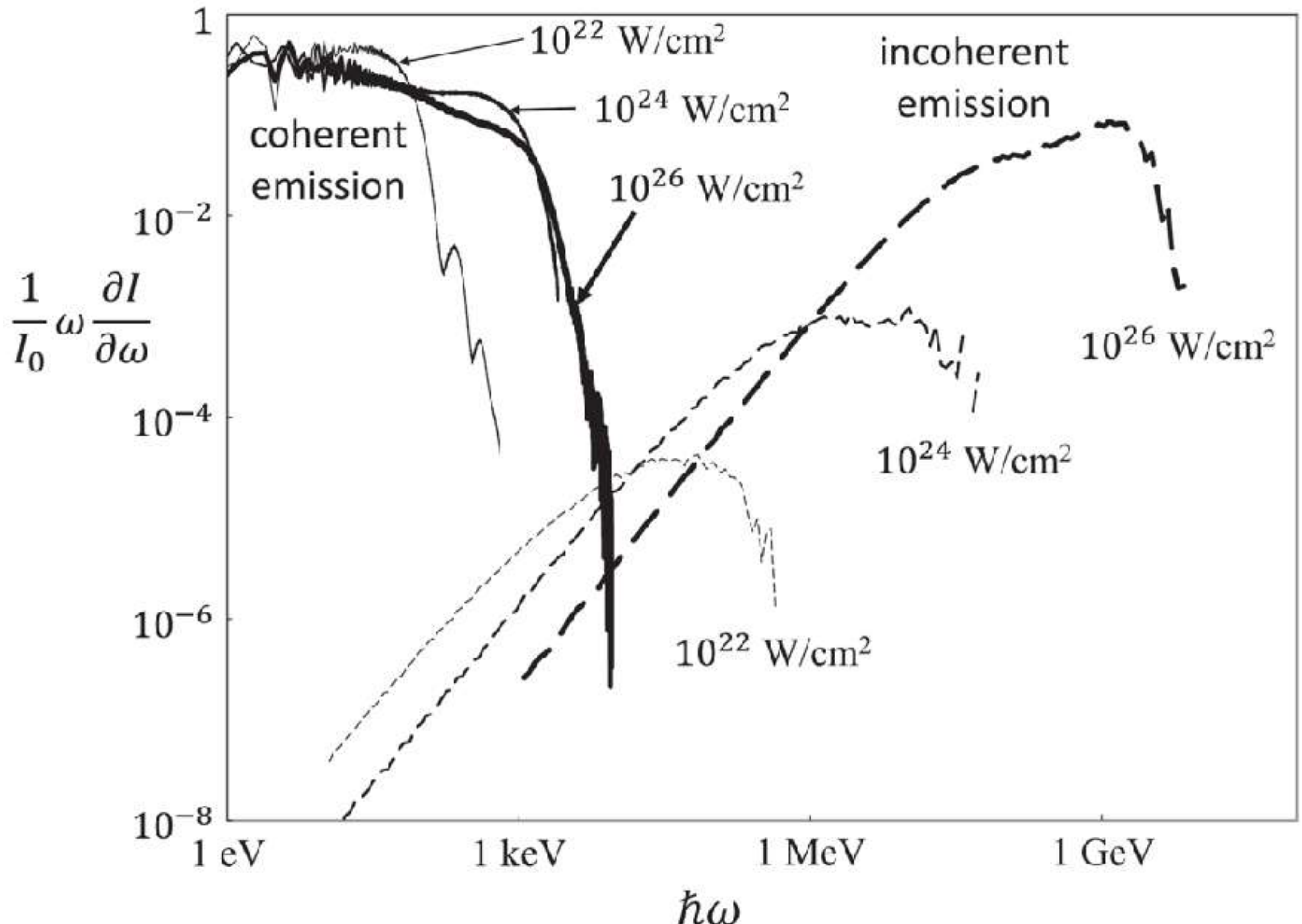
- Classical radiation reaction: $I \sim 10^{22}$ W/cm^2
- Quantum radiation reaction: $I \geq 10^{23}$ W/cm^2

Double counting?



A. Gonoskov et al., PRE 92 (2), 023305 (2015)

Interaction between laser and near-critical target



A. Gonoskov et al., PRE 92 (2), 023305 (2015)

Synchrotron emission

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- Making inclusion of synchrotron emission via direct solving of Maxwell's equations impossible in PIC scheme. *Cannot achieve good enough grid resolution.*
- Luckily the power spectra helps us.
- Assuming photons emitted along direction of propagation:

$$\mathbf{f}_{RR}^{cl} = -\frac{2}{3}\frac{e^2m^2c}{\hbar^2}\chi^2\mathbf{v}. \qquad \chi = \gamma\frac{H_{\text{eff}}}{E_S}$$

- Same as ultra-relativistic limit of the Landau-Lifshitz expression.
- Postprocessing: cannot be executed in real time, no feedback.
- On-the-fly: neglect interference between time-steps. Only a small error in regime where radiation reaction important.

Radiation reaction

- **Extensive literature** (not updated!) on this topic, as well as many different implementations, see, e.g.,

Di Piazza, Lett. Math. Phys. **83**, 305 (2008)
Bell & Kirk, PRL **101**, 200403 (2008)
Di Piazza et al., PRL **105**, 20403 (2010)
Bulanov et al., PoP **17**, 063102 (2010)
Sokolov et al., PoP **18**, 093109 (2011)
Thomas et al., PRX **2**, 041004 (2012)
Di Piazza et al., RMP **84**, 1177 (2012)
Schlegel & Tikhonchuk, NJP **14**, 073034 (2012)
Chen et al., PRSTAB **16**, 030701 (2013)
Mackenroth et al., PPCF **55**, 124018 (2013)
Ilderton & Torgrimsson, PRD **88**, 025021 (2013)
Ridgers et al., J. Comp. Phys. **260**, 273 (2014)
Yoffe et al., NJP **17**, 053025 (2015)
Tamburini et al., PRE **89**, 021201 (2014)
etc…

Radiation reaction/friction

- Classical radiation reaction described using Lorentz-Abraham-Dirac (LAD) theory

$$m\dot{u}^\mu = eF^{\mu\nu}u_\nu - \frac{2}{3}\frac{e^2}{4\pi}(u^\mu \ddot{u}^\nu - u^\nu \ddot{u}^\mu)$$

- or the perturbative expansion (lacking runaways etc.) due to Landau & Lifshitz (LL)

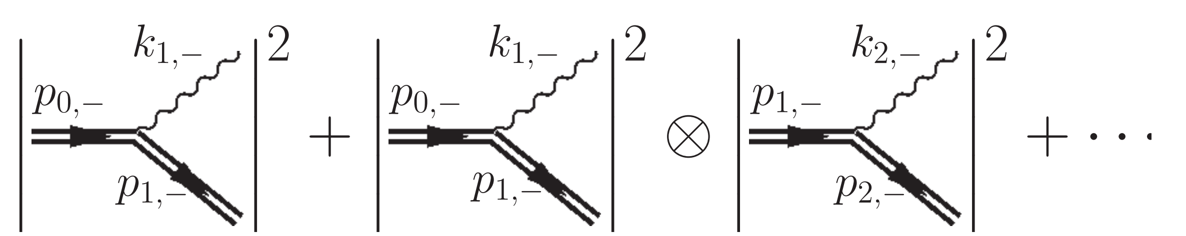
$$\dot{u}^\mu = \frac{e}{m}F^{\mu\nu}u_\nu + \frac{2}{3}\frac{e^2}{4\pi}\left\{\frac{e}{m^2}\dot{F}^{\mu\nu}u_\nu + \frac{e^2}{m^3}F^{\mu\alpha}F_\alpha{}^\nu u_\nu - \frac{e^2}{m^3}u_\alpha F^{\alpha\nu}F_\nu{}^\beta u_\beta\, u^\mu\right\}$$

- Works in classical regime when

$$\chi \equiv \frac{e\hbar\sqrt{(F^{\mu\nu}u_\nu)^2}}{m^2c^4} \ll 1, \qquad \Longrightarrow \qquad \hbar a_0\gamma\omega \ll mc \quad (a_0 = eE/\omega mc)$$

- QED regime when

$$\chi \sim 1 \qquad \dot{u}^\mu = \frac{e}{m}F^{\mu\nu}u_\nu + \frac{d^2P}{dt\,d\chi_\gamma}$$



Burton & Noble, Contemporary Phys. **55**, 110 (2014); Di Piazza et al., RMP **84**, 1177 (2012)

Radiation reaction: which model?

- There are a number of classical models in the literature (see also Vranic et al., Comp. Phys. Comm. **204**, 141 (2016)).

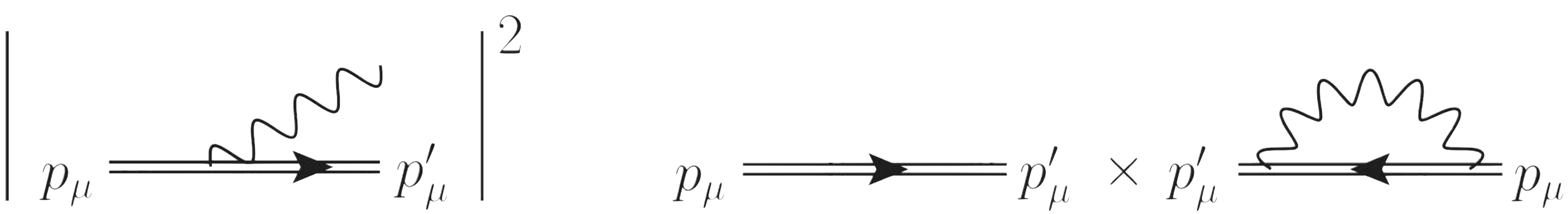
$$\ddot{x}^\mu = f^{\mu\nu}\dot{x}_\nu + \frac{2}{3}\frac{e^2}{4\pi m}R_{\mu\nu}\dot{x}^\nu$$

Radiation Reaction	$R_{\mu\nu}$
Abraham Lorentz Dirac (LAD)	$\dddot{x}\dot{x} - \dot{x}\dddot{x}$
Landau Lifshitz (LL)	$\dot{f} + (f^2\dot{x})\dot{x} - \dot{x}(f^2\dot{x})$
Eliezer Ford O'Connell (EFO)	$\frac{\mathrm{d}}{\mathrm{d}\tau}(f\dot{x})\dot{x} - \dot{x}\frac{\mathrm{d}}{\mathrm{d}\tau}(f\dot{x})$
Mo and Papas (MP)	$(f\ddot{x})\dot{x} - \dot{x}(f\ddot{x})$
Herrera (H)	$(f^2\dot{x})\dot{x} - \dot{x}(f^2\dot{x})$
Sokolov (S)	$q \mp m\dot{x}$

A. Ilderton & G. Torgrimsson, PRD and PRB (2013)

Radiation reaction: which model?

- There are a number of classical models in the literature (see also Vranic et al., Comp. Phys. Comm. **204**, 141 (2016)).
- Start from QED, take the classical expansion to order e^2



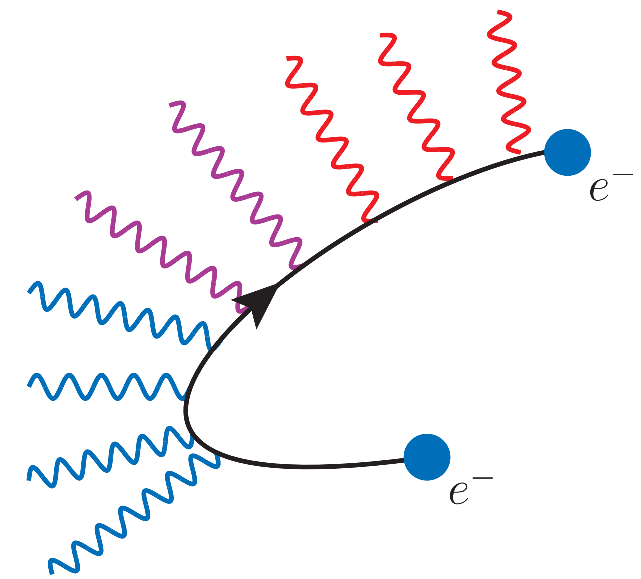
Radiation Reaction	$\mathcal{O}(e^2)$	$\mathcal{O}(e^4)$
Abraham Lorentz Dirac (LAD)	✓	?
Landau Lifshitz (LL)	✓	?
Eliezer Ford O'Connell (EFO)	✓	?

To this order, we could (in principle) distinguish between LAD, LL, EFO

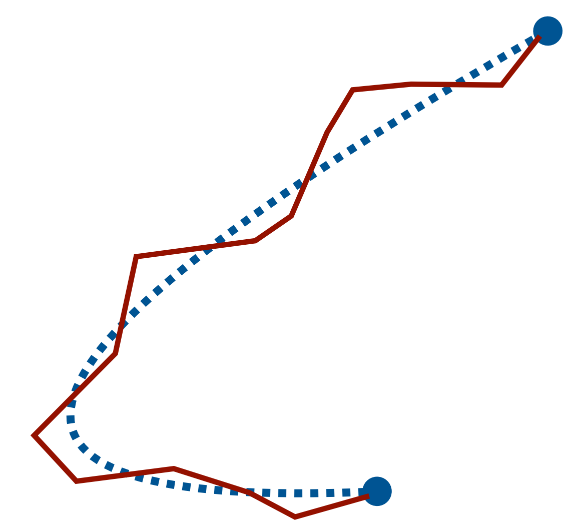
A. Ilderton & G. Torgrimsson, PRD and PRB (2013)

Radiation reaction: short summary

- Accelerated charged particles emit radiation.
- Strong enough accelerating field: emitted radiation will give significant momentum kick to electron.
- Momentum kick: radiation reaction.*

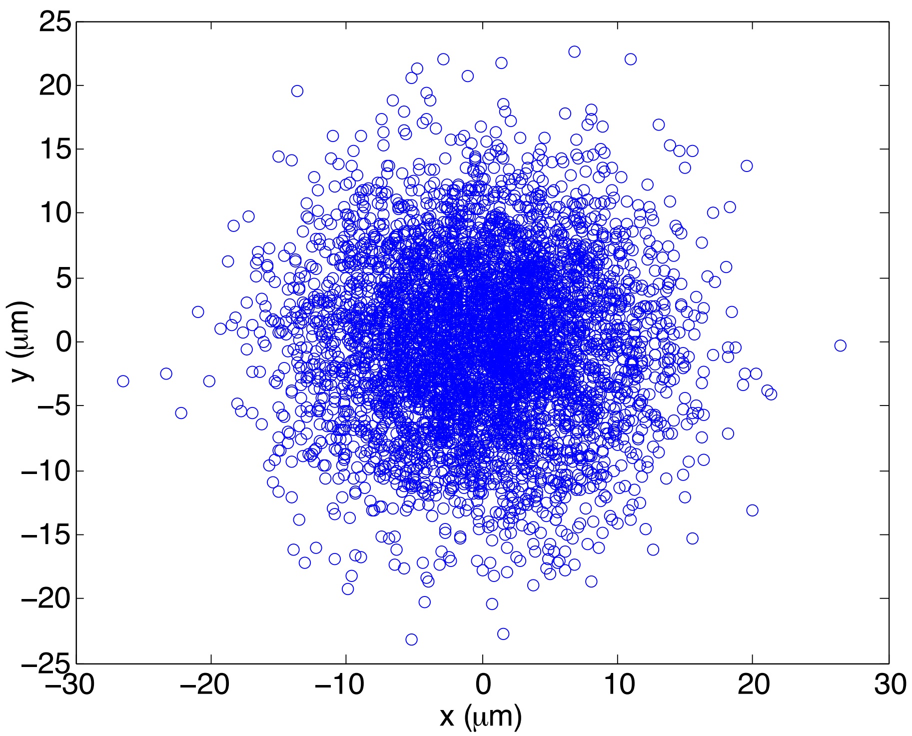


- Classical regime: continuous emission, Landau-Lifshitz description.
- Quantum regime: discrete, stochastic emission.
- Straggling‡: stochastic emission makes electron reach classically forbidden regions of laser pulse.

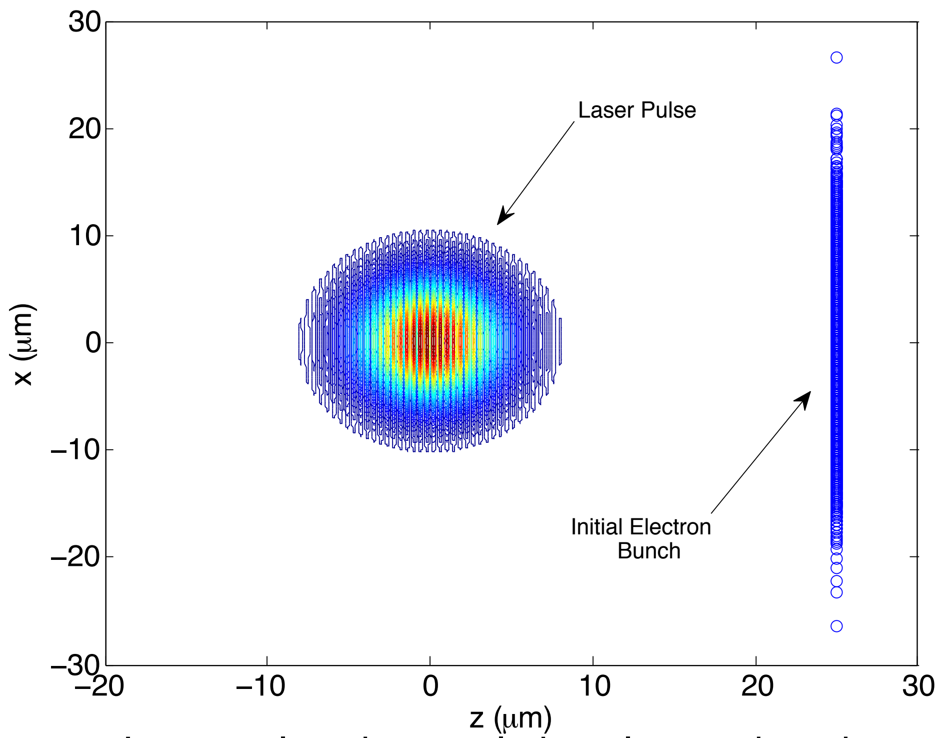


*Di Piazza et al., Rev. Mod. Phys. **84**, 1177 (2012)
‡Shen & White, Phys. Rev. Lett. **28**, 455 (1972)
‡Blackburn et al., Phys. Rev. Lett. **112**, 015001 (2014)

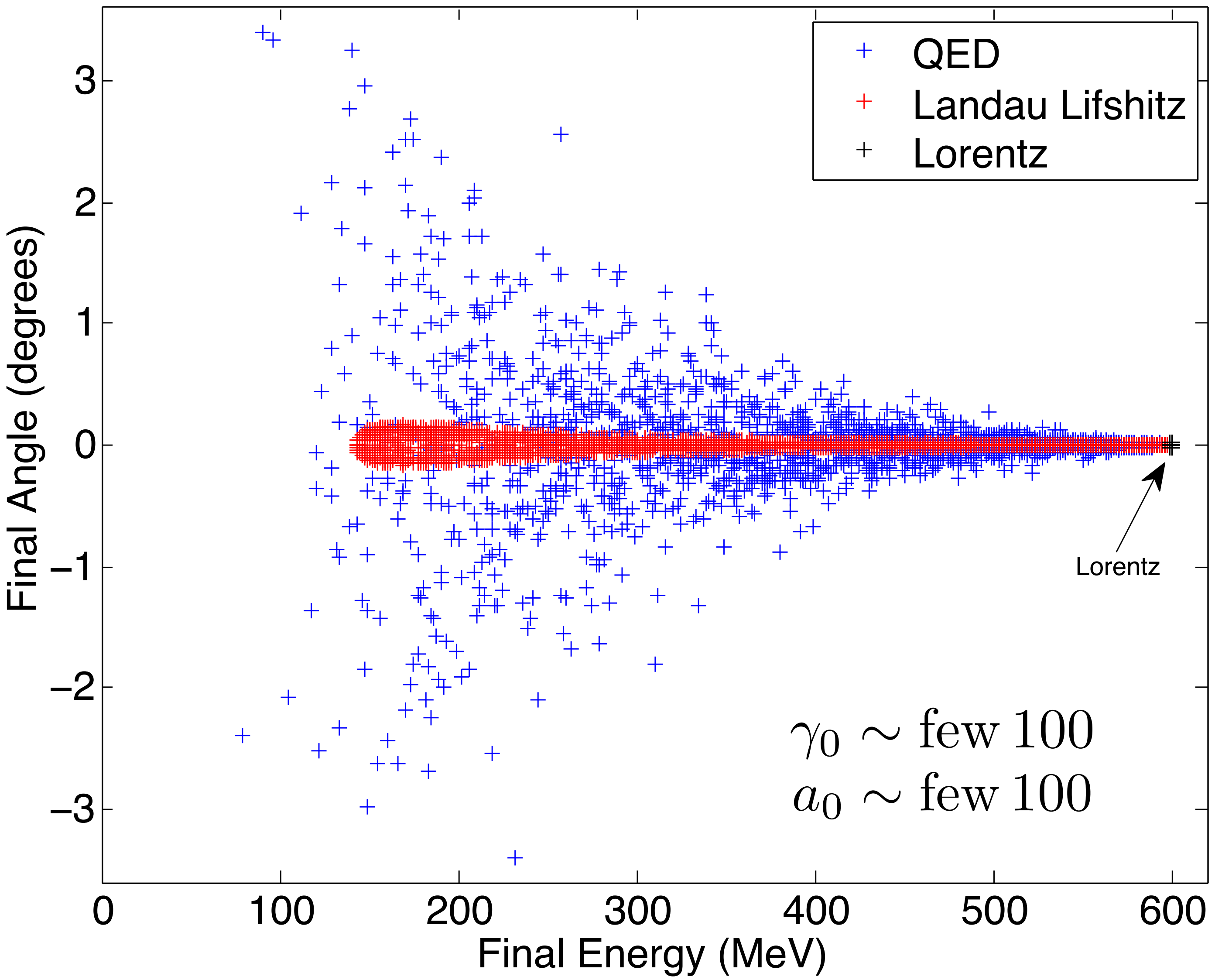
Radiation reaction: electron distribution



Initial spatial spread of electron bunch



Laser pulse shape relative electron bunch



Final angular distribution of electrons

Green & Harvey, PRL **112**, 164801 (2014)
Li, Hatsagortsyan, & Keitel, PRL **113**, 044801 (2014)
Green & Harvey, Comp. Phys Comm. **192**, 313 (2015)
Harvey, Ilderton, & King, Phys. Rev. A **91**, 013822 (2015)
Harvey, Marklund & Wallin, Phys. Rev. A **93**, 022112 (2016)

Radiation reaction: radiation distribution

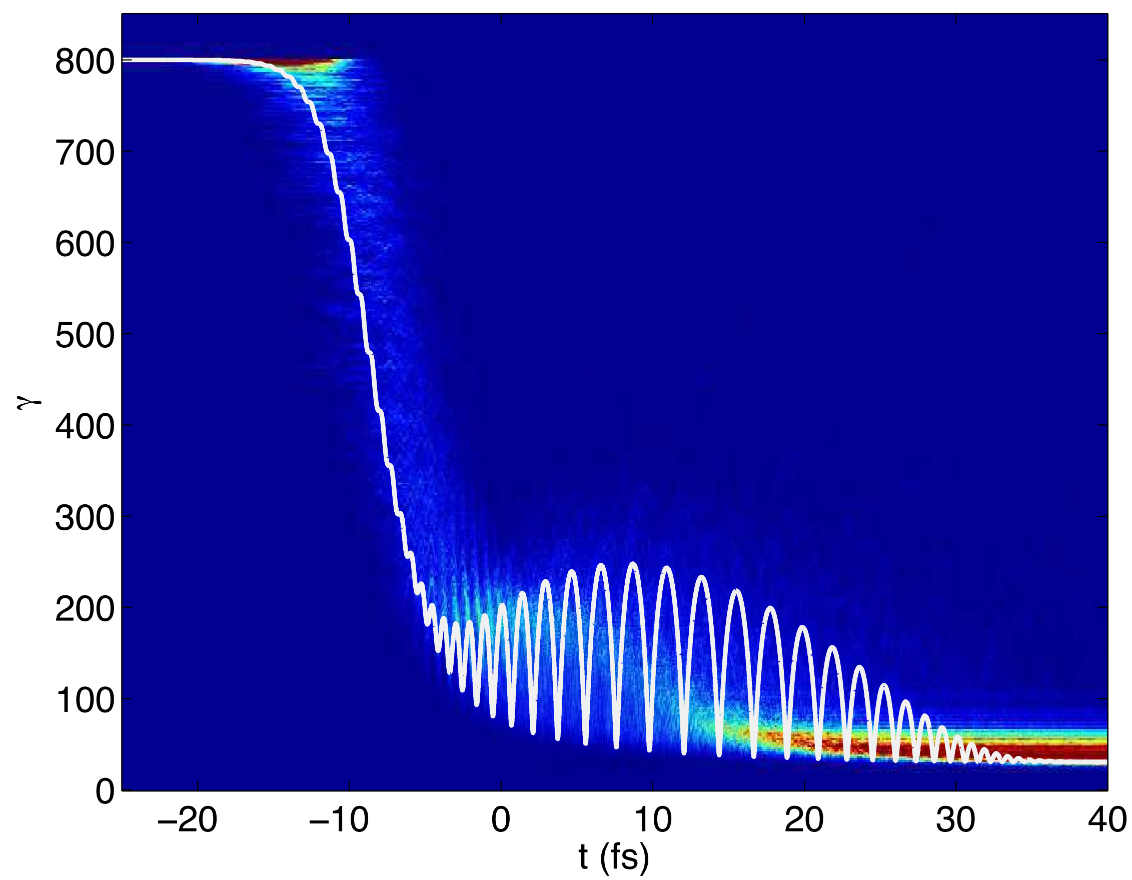


FIG. 4. Density plot showing how the electron γ-factor changes with time (statistical distribution generated by recording the paths of 500 QED electrons all with the same initial condition $\gamma_0 = 800$). Parameters are $a_0 = 200$, $\lambda = 0.8\mu$m and duration 30fs. The white line shows the γ-factor for a classical electron.

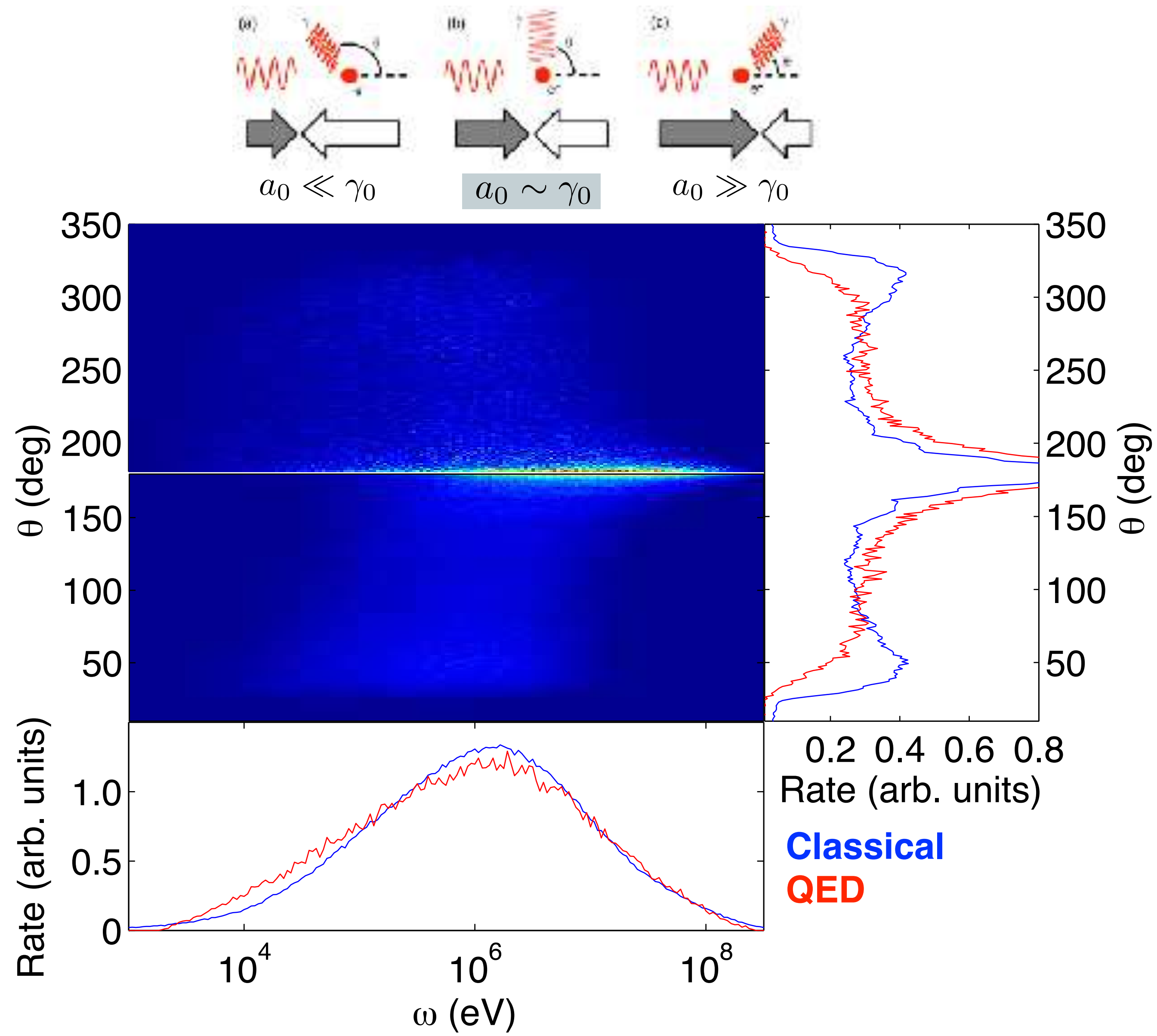


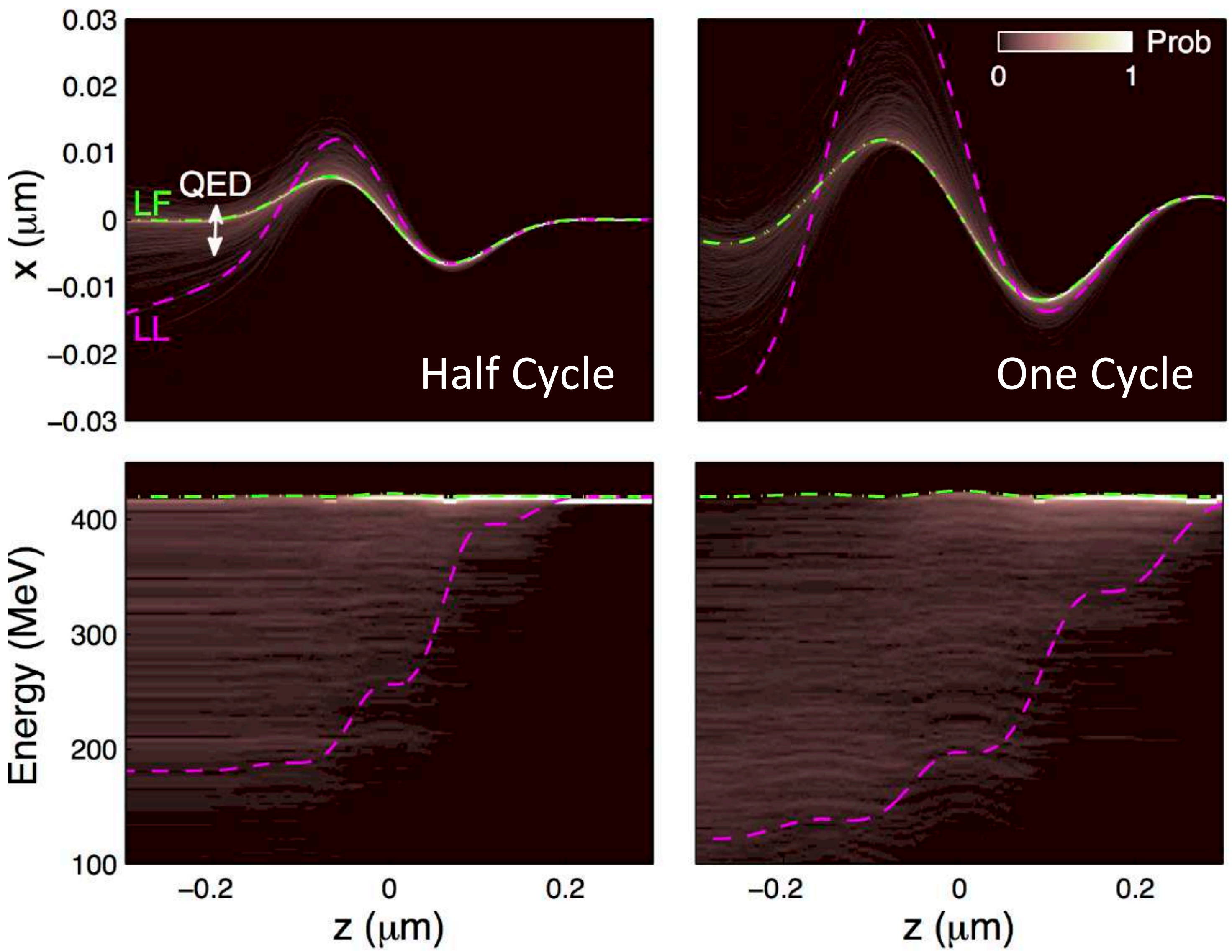
FIG. 6. Emission spectrum for the realistic case ($a_0 = 250$) described in Fig. 5. The centre panel shows the radiation intensity as a function of frequency and angle. This panel is split into two, the top half showing the emissions for the QED simulation and the bottom half the classical. The right hand panel shows the total angular rate summed over all frequencies (both classical and QED for all angles), and the bottom panel the total frequency rate summed over all angles. Red lines: QED. Blue lines: classical.

Harvey, Marklund & Wallin, Phys. Rev. A **93**, 022112 (2016)
Vranic, Martins, Vieira, Fonseca & Silva, PRL **113**, 134801 (2014)
Green & Harvey, PRL **112**, 164801 (2014)
Neitz & Di Piazza, PRL **111**, 054802 (2013)
Li, Hatsagortsyan & Keitel, PRL **113**, 044801 (2014)
Harvey, Heinzl & Ilderton, PRA **79**, 063407 (2009)

Quantum quenching

- Motion without emission ↔ motion according to Lorentz force.

Direction of electron motion



Top panel shows electron (420 MeV) trajectories in a collision with a laser pulse (a_0 = 200) for 1000 single electron QED simulations. Left panel is for a 1.4 fs pulse, right is for a 2.7 fs pulse.

Lower panel shows energy loss when the electrons pass through the pulse. Clear signal of quantum behavior in such an ideal case. Purple curves show shows classical radiation reaction.

Harvey et al., PRL 118, 105004 (2017)

Including classical radiation reaction in PIC codes

- PIC schemes work with super particles where the charge to mass ratio is kept fixed for each species *q/m.*

- This works because the acceleration due to the Lorentz force depends on *q/m*.

- Not true for RR force. However →

Let us consider a macro particle that represents η electrons. The charge of the macro particle is $e_m = \eta e$, and the mass is $m_m = \eta m_e$. For a single particle with the same mass and charge as the macro particle, the radiation reaction would be η times stronger than in the case of a single electron:

$$\frac{F_{RR}}{F_L} \propto \frac{(\eta e)^3}{(\eta m_e)^2} = \eta \frac{e^3}{m_e^2} \tag{12}$$

and the trajectory of such particle would be different than the trajectory of a single electron (Fig. 1). This result would be equivalent to assuming that η electrons are radiating coherently. As a consequence, the results of a PIC simulation would be qualitatively different for different number of particles per cell or different cell sizes. To obtain the correct dynamics of a macro-particle, it is therefore essential to use the real charge and mass to calculate the correct radiation reaction coefficient for a particular particle species. This approach yields the same result regardless of the macro-particle weight.

M. Vranic et al. Comput. Phys. Commun. 204, 141-151 (2016)

Radiative trapping

Different trapping mechanisms and dynamics in strong fields:

A. Relativistic chaos (Bauer, Mulser, Steeb, PRL 75 (1995))

B. Relativistic reversal (Kaplan, Pokrovsky, PRL 95 (2005))

C. Normal radiative trapping (Kirk, Bell, Arka, PPCF 51 (2009))

D. Ponderomotive trapping (Lehmann, Spaschek, PRE 85 (2012))

E. *Anomalous radiative trapping*: electrons trapped at peak electric field. (A. Gonoskov et al., Phys. Rev. Lett. **113**, 014801 (2014)).

Anomalous radiative trapping

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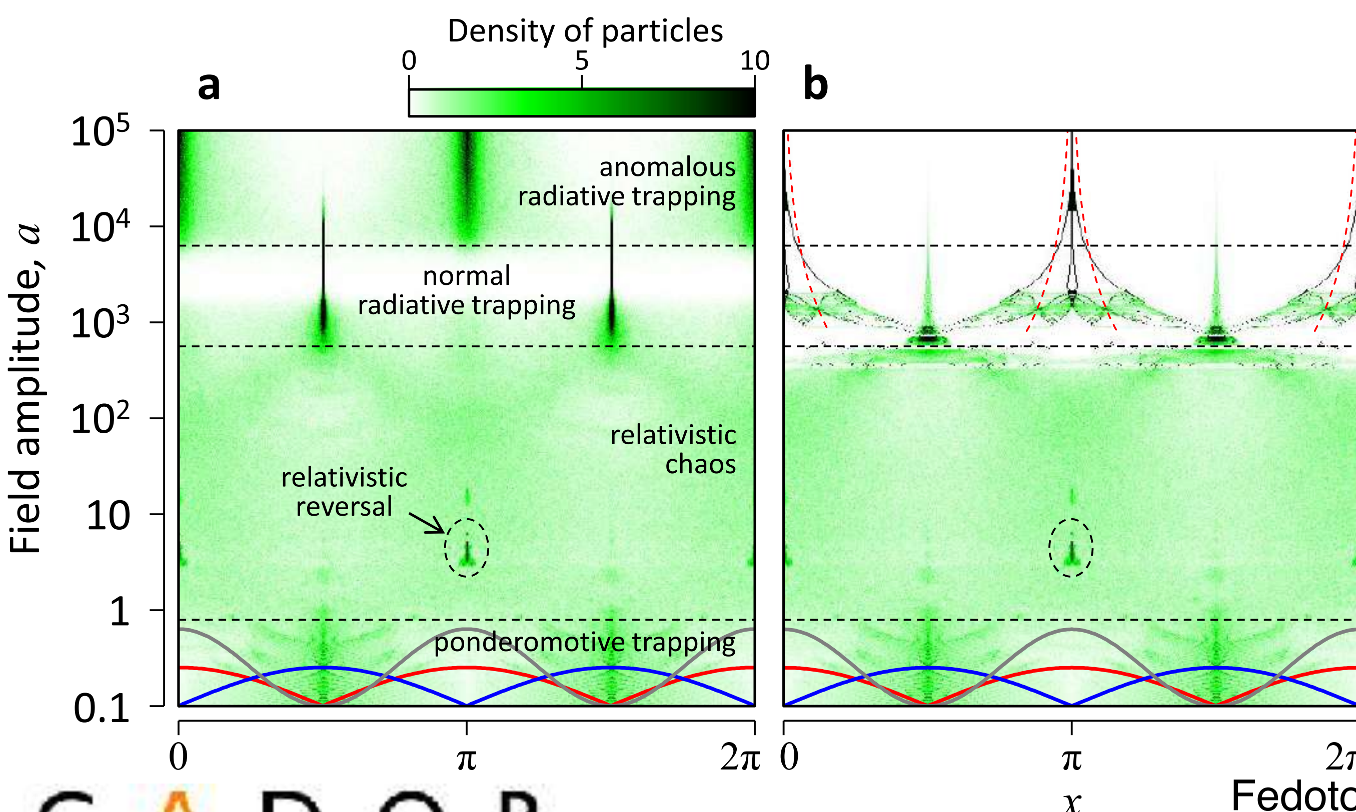
E. *Anomalous radiative trapping*: electrons trapped at peak standing wave electric field. (A. Gonoskov et al., Phys. Rev. Lett. **113**, 014801 (2014)).



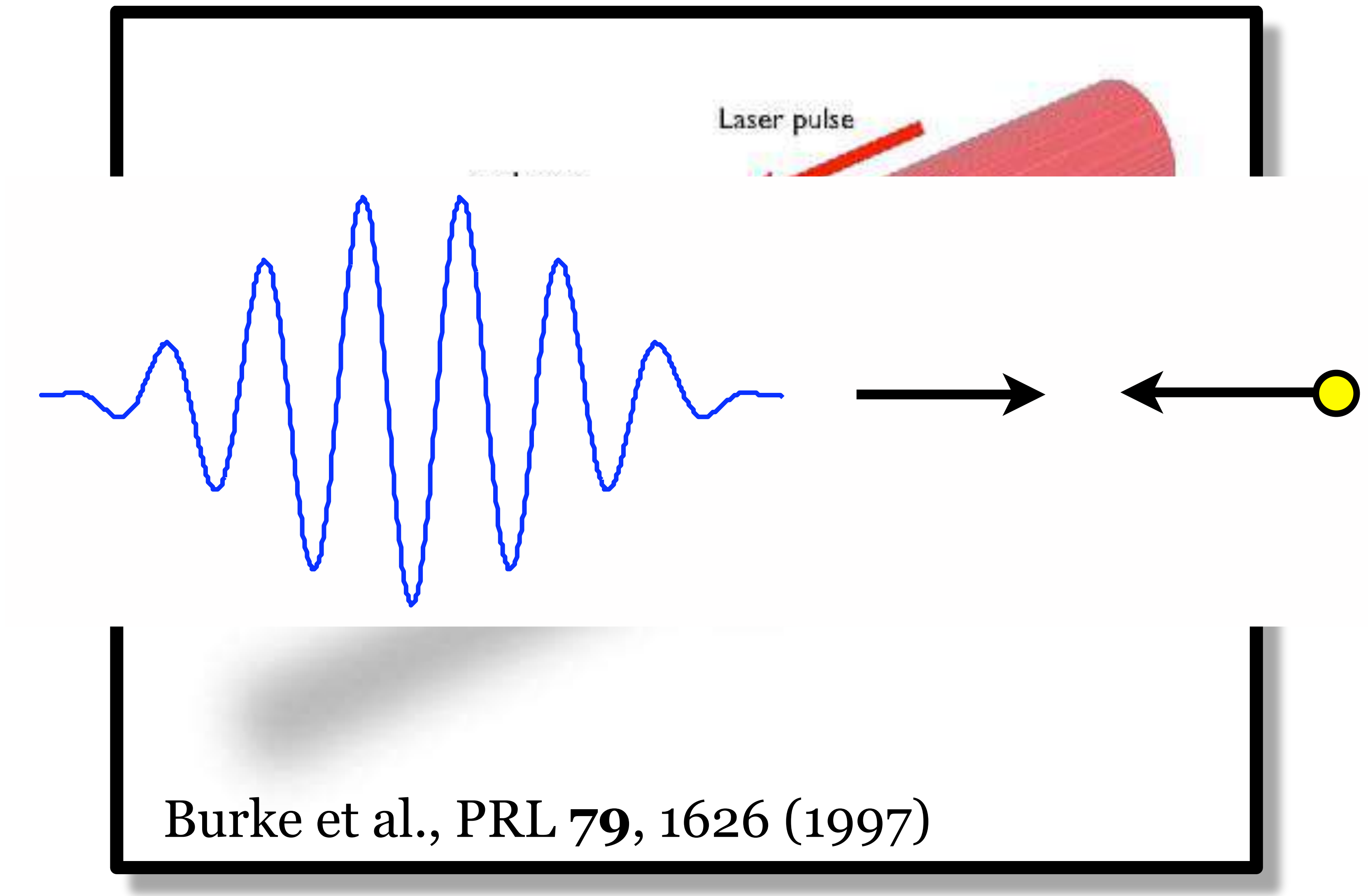
Electron density in e-dipole field
Photons with energy > 3 GeV in cyan

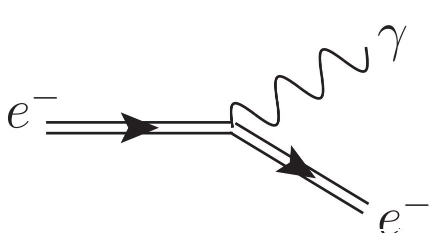
P I C A D O R

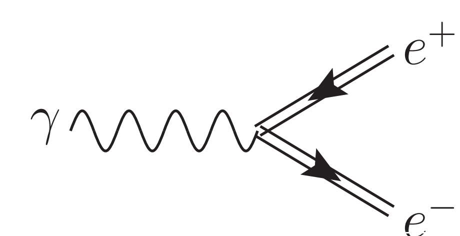
Fedotov, Elkina, Gelfer, Narozhny & Ruhl, PRA **90**, 053847 (2014)
Ji, Pukhov, Kostyukov, Shen & Akli, PRL **112**, 145003 (2014)

Compton scattering & the Breit—Wheeler process

- Perfect testbed for single particle effects! Lots of interesting physics! (Bula et al., 1996; Bamber et al., Phys. Rev. D (1999)).







Nonlinear Compton scattering

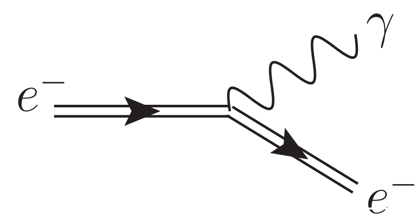
$$e + n\omega \to e' + \gamma$$

Multi-photon Breit-Wheeler scattering

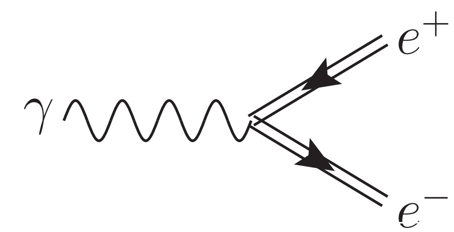
$$\gamma + n\omega \to e^+ + e^-$$

Multi-photon processes in intense fields

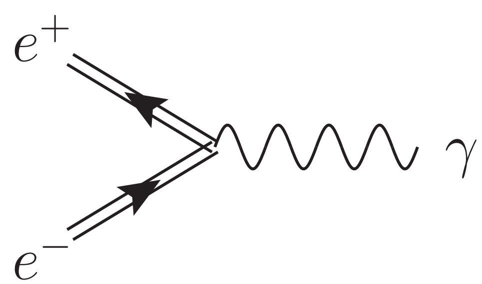
- **Lowest order processes**
  - Nonlinear Compton scattering



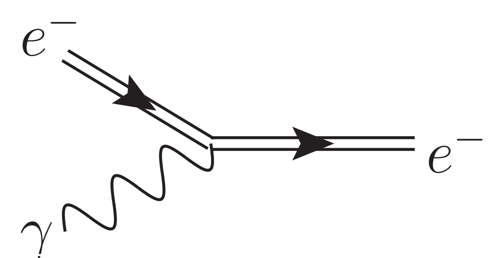
  - Pair production



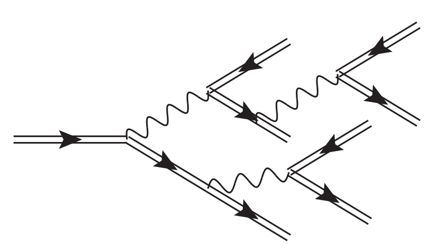
  - Pair annihilation



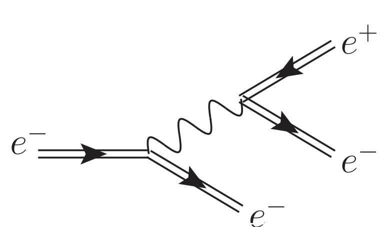
  - One photon absorption



- **Related processes (see talk by F. Mackenroth)**
  - Cascading (on-shell trident)



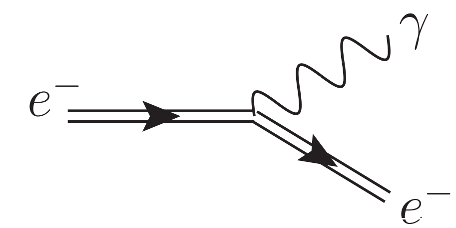
  - Trident



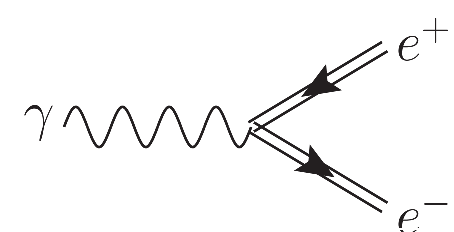
Multi-photon emissions: in high intensity fields (*a* >> 1), multi-photon emission factorized into several single photon events. Captured via synchrotron models. For high energy electrons at low intensity (*a* ~ 1) significant low-frequency contribution from off-shell channels. (Seipt & Kämpfer, PRD **85**, 1019701 (2012); Mackenroth & Di Piazza, PRL **110**, 070402 (2013)). For double nonlinear Compton scattering, see King, Phys. Rev. A 91, 033415 (2015).

See also: Hu et al., PRL **105**, 080401 (2010); Ilderton, PRL **106,** 020404 (2011); King & Ruhl, PRD **88**, 013005 (2013)

Nonlinear Compton scattering and pair production



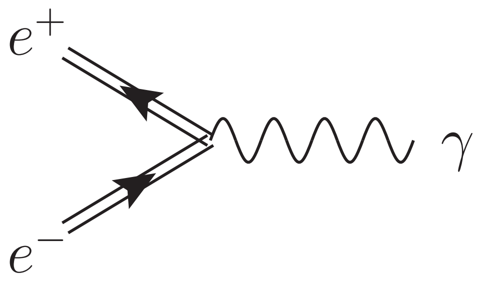
- **Nonlinear Compton scattering:**
  - No energy or intensity threshold for channel.
  - Low energy limit: matches spectrum from classical particle with Lorentz force.
  - Classical limit of electron recoil: radiation reaction.



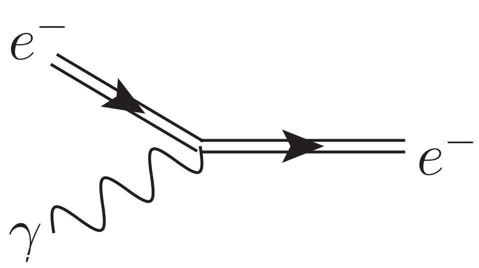
- **Stimulated pair production:**
  - Threshold process: probability vanishes in low energy/classical limit.
  - For highly relativistic intensities, the process of pair production takes very small energy from laser as compared to the acceleration of the produced pairs.
  - Thus, the number of pairs needs to be prolific before pair production takes significant energy from laser pulse.

Pair annihiliation and one photon absorption

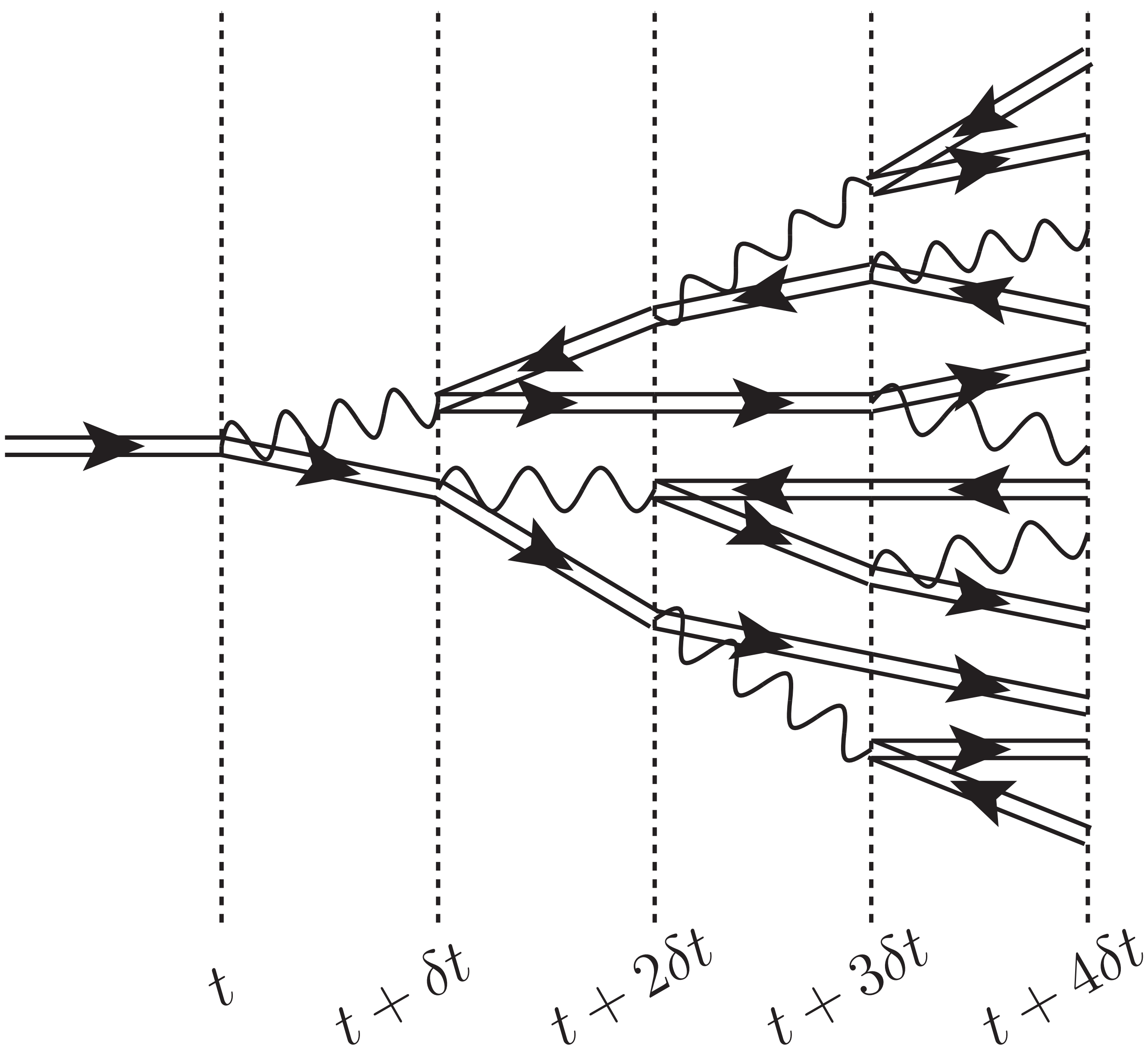
- **Pair annihilation to one photon:**
  - Suppressed by infinite volume factor as compared to the crossing symmetry diagrams above.
  - Energy-momentum conservation —> single four-momentum of photon. final phase space collapses to single point.
  - *Of importance for approach to equilibrium, or high density systems.*



- **One photon absorption:**
  - Suppressed as above.



Cascading



Simplified picture of numerical implementation of cascading in PIC code. All particles real. Including trident is a challenge (see talk by F. Mackenroth). For a discussion of double nonlinear Compton scattering, see King, Phys. Rev. A 91, 033415 (2015).

Ritus (1972); Bell & Kirk, PRL (2008) ; Ilderton, PRL (2010); Klepikov (1964); Nikishov & Ritus (1964); Elkina et al. (2011)

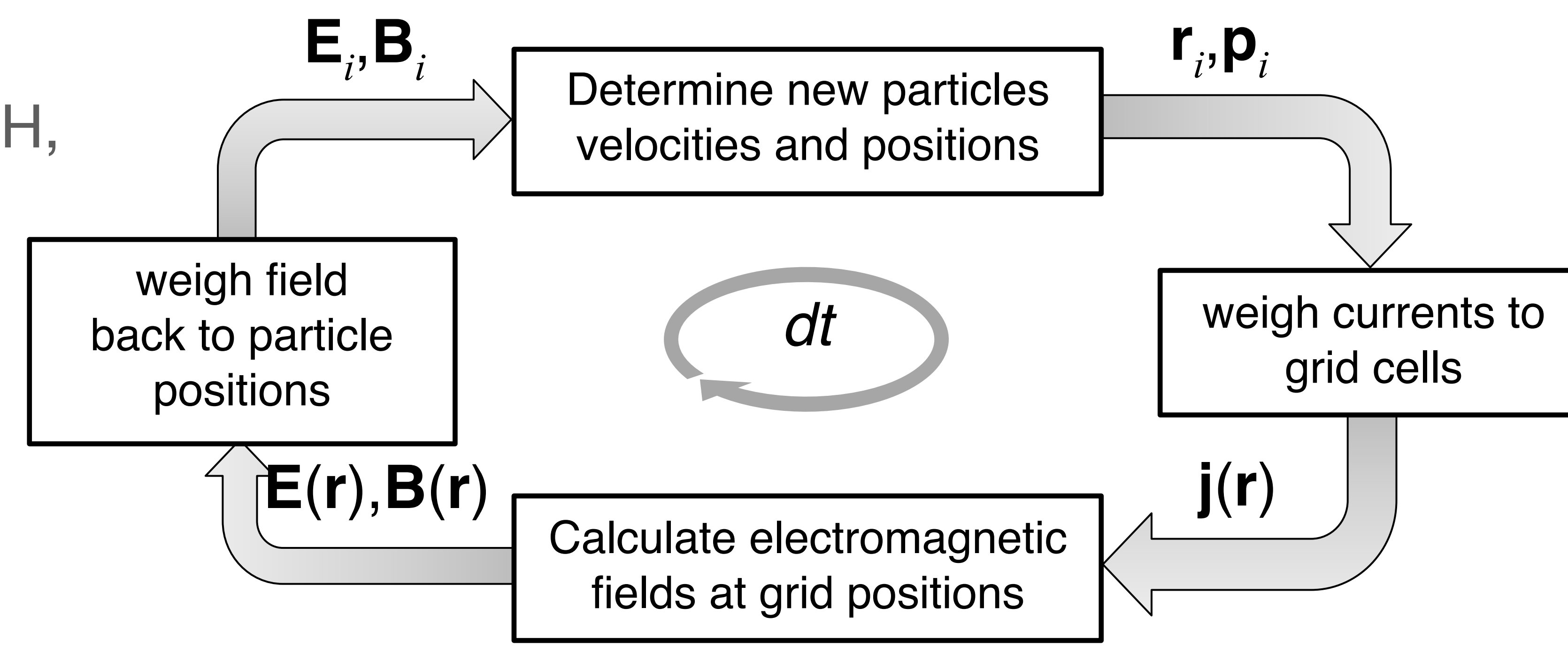
Classical particle-in-cell scheme

- The particles interact via solution of Maxwell's equations on discrete grid.
- The ensemble of particles are represented by a smaller number of *super-particles*, keeping the mass-to-charge ratio of the real particles.
- Super-particles have distributed charges and generate distributed currents with the size of order Δx.
- Many different codes: EPOCH, OSIRIS, PICADOR, ELMIS, PIConGPU…

$$\Delta x \sim \lambda_D = \frac{v}{\omega_p} = \sqrt{\frac{T}{8\pi e^2 N}} \quad \text{(Debye length)}$$

$$dt \sim T_p/4, \quad T_p = 2\pi/\omega_p$$

$$\omega_p = (4\pi e^2 N_e/m)^{1/2}$$



QED processes in PIC codes

- No trajectories. Instead, scattering probabilities via **Monte Carlo modules.**

1. Calculate the scattering probability in locally constant crossed fields,

$$E^2 - B^2 = \mathbf{E} \cdot \mathbf{B} = 0$$

for given event (Nikishov & Ritus, JETP **19**, 529 + 1191(1964); Ritus, J. Sov. Laser Res. **6**, 497 (1985)).

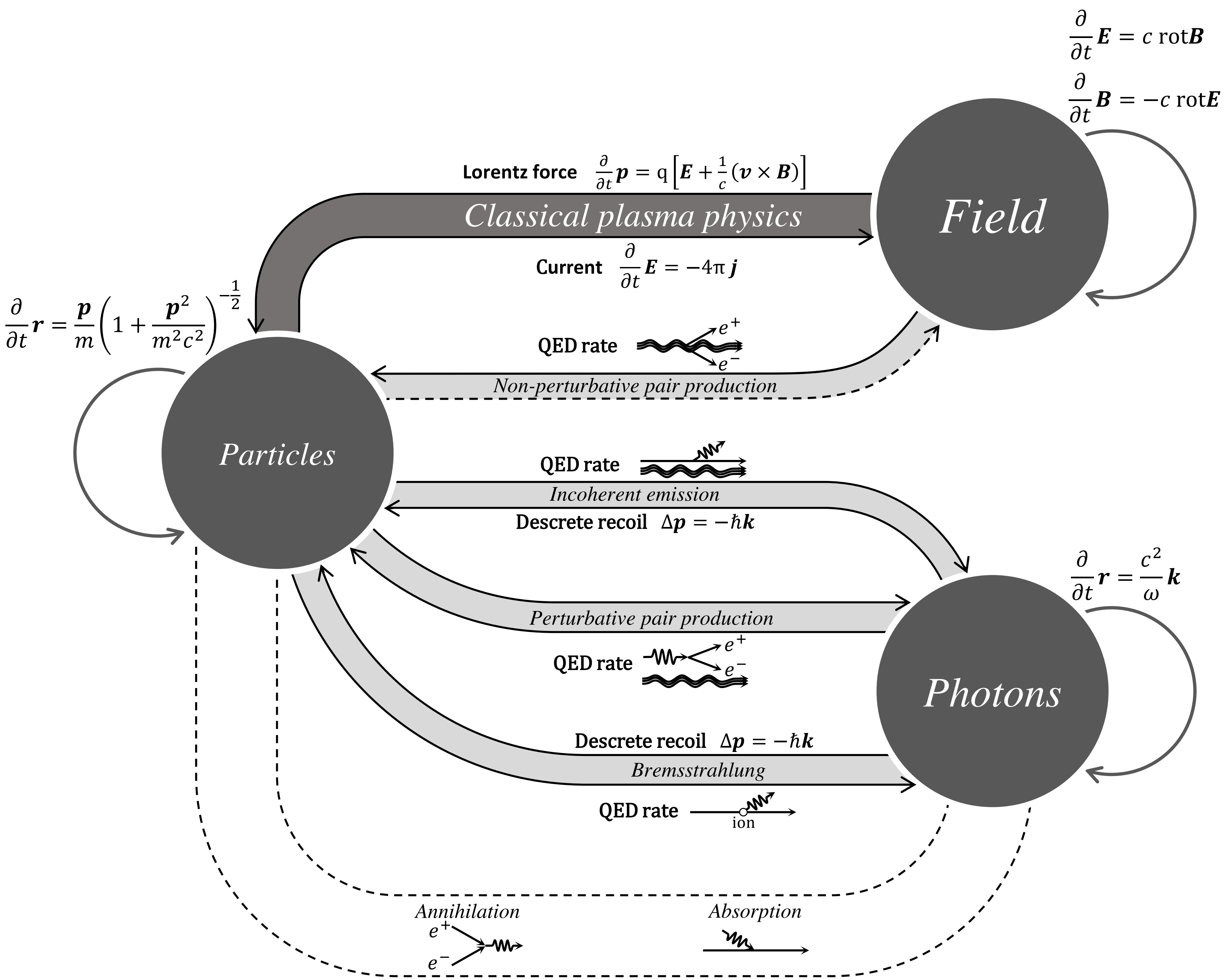
3. Divide by infinite interaction time to obtain finite rate.

4. Rate assumed to be *local* transition rate using locally constant field approximation.

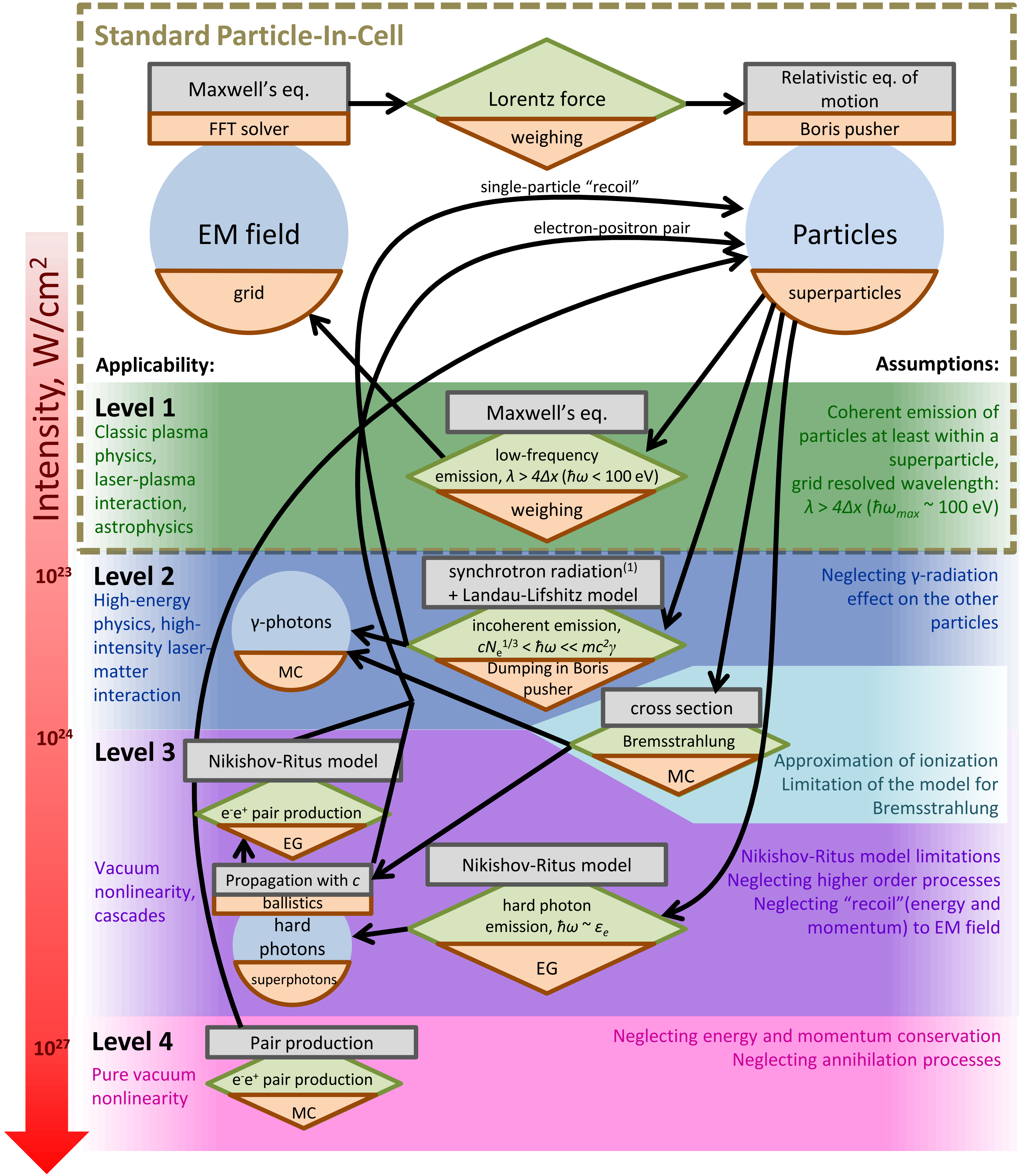
5. After time step Δ*t* combine rate with statistical event generator, adding or removing particle species as appropriate.

6. Because number of pairs can grow exp, we need a thinning or merging algorithm for the particles. This keeps the particle number on a computational level.

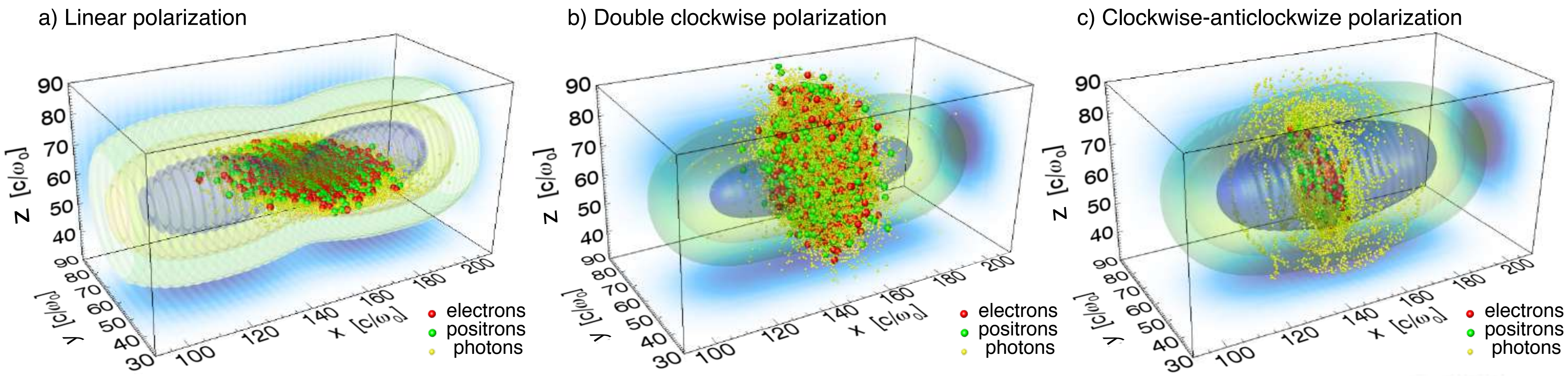
QED in particle-in-cell codes



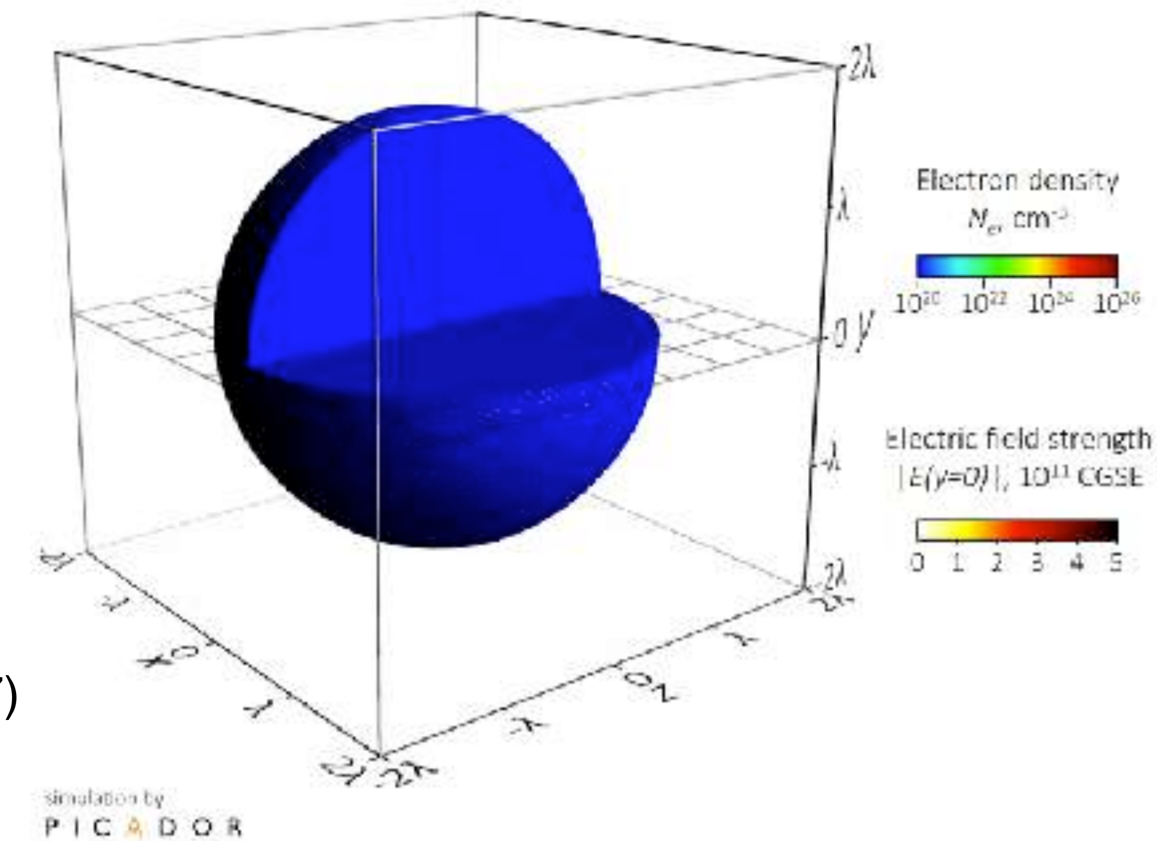
C. Ridgers et al., J. Comp. Phys. 260, 273 (2014); T.D. Arber et al., Plasma Phys. Control. Fusion 57, 113001 (2015); A. Gonoskov et al., Phys. Rev. E **92**, 023305 (2015); M. Vranic et al., Comp. Phys. Comm. 204, 141 (2016)


Standard Particle-In-Cell
Maxwell's eq.
FFT solver
Lorentz force
weighing
Relativistic eq. of motion
Boris pusher
EM field
grid
Particles
superparticles
single-particle "recoil"
electron-positron pair
Intensity, W/cm2
Applicability:
Assumptions:
Level 1
Classic plasma physics, laser-plasma interaction, astrophysics
Maxwell's eq.
low-frequency emission, λ > 4Δx (ħω < 100 eV)
weighing
Coherent emission of particles at least within a superparticle, grid resolved wavelength: λ > 4Δx (ħωmax ~ 100 eV)
10^23
Level 2
High-energy physics, high-intensity laser-matter interaction
γ-photons
MC
synchrotron radiation(1) + Landau-Lifshitz model
incoherent emission, cNe1/3 < ħω << mc2γ
Dumping in Boris pusher
Neglecting γ-radiation effect on the other particles
cross section
Bremsstrahlung
MC
Approximation of ionization
Limitation of the model for Bremsstrahlung
10^24
Level 3
Nikishov-Ritus model
e-e+ pair production
EG
Propagation with c
ballistics
hard photons
superphotons
Vacuum nonlinearity, cascades
Nikishov-Ritus model
hard photon emission, ħω ~ εe
EG
Nikishov-Ritus model limitations
Neglecting higher order processes
Neglecting "recoil"(energy and momentum) to EM field
10^27
Level 4
Pure vacuum nonlinearity
Pair production
e-e+ pair production
MC
Neglecting energy and momentum conservation
Neglecting annihilation processes

Seeded cascades in colliding pulses: examples



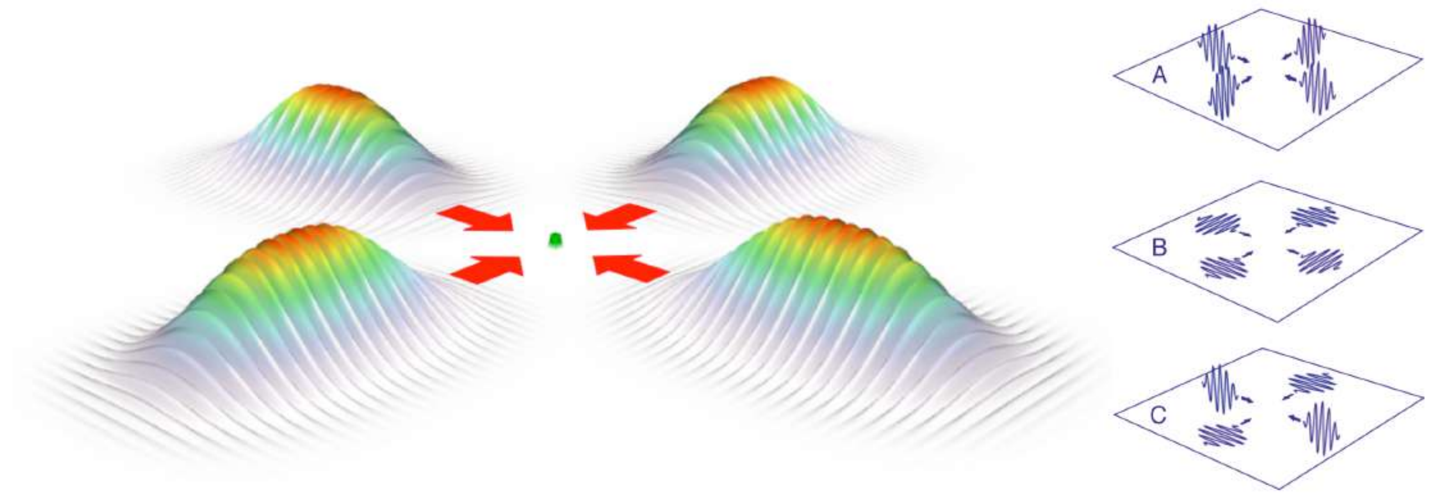
Grismayer et al., Phys. Rev. E 95, 023210 (2017)



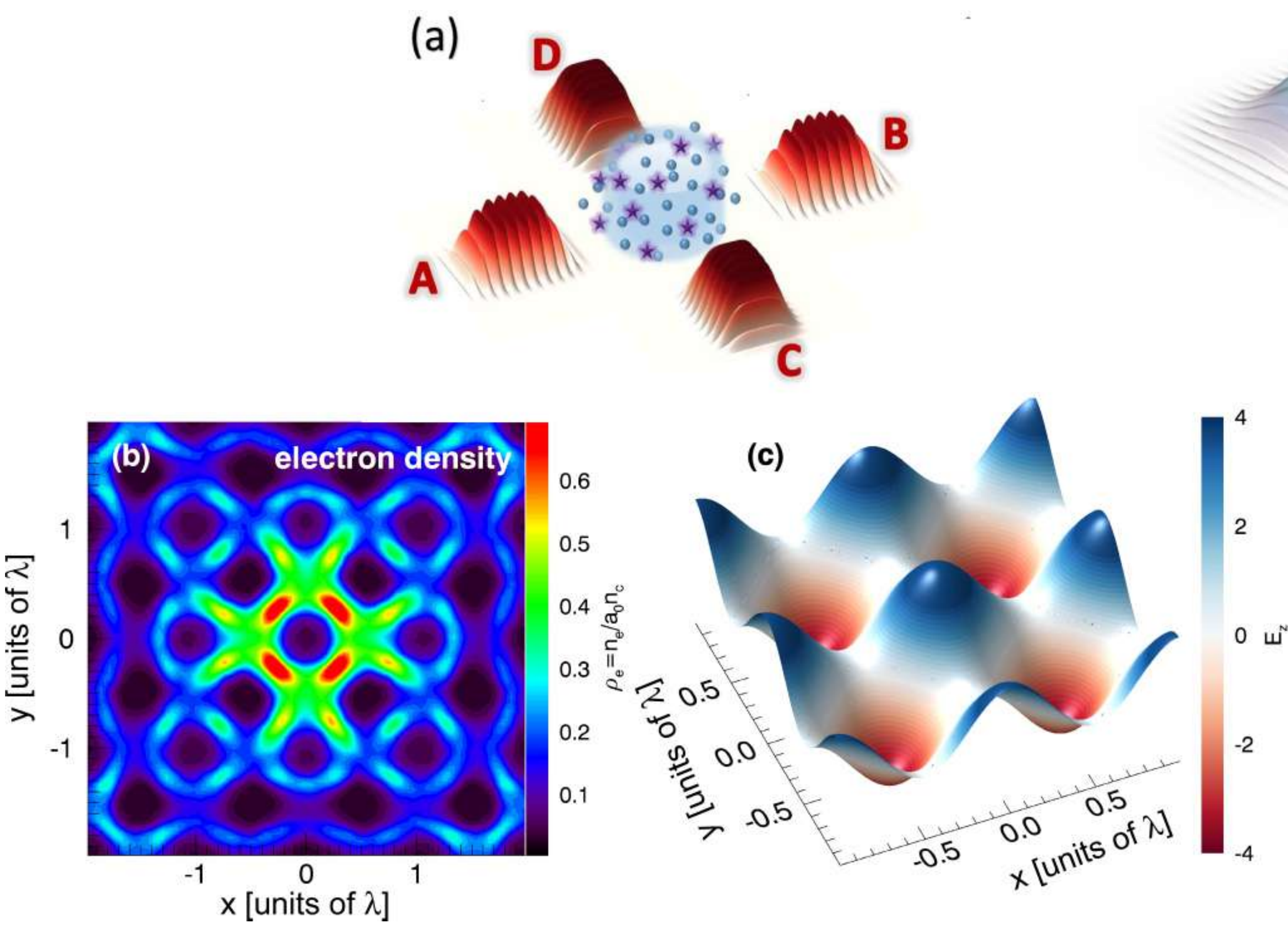
Gonoskov et al., PRX, in press (2017)

Using QED PIC simulations, processes involving radiation reaction, synchrotron emission, Breit—Wheeler pair production, and collective plasma effects can be analysed in a self-consistent way.

Seeded cascades in colliding pulses: examples



Vranic et al., PPCF 59, 014040 (2017)



Gong et al., PRE 95, 013210 (2017)

Using QED PIC simulations, processes involving radiation reaction, synchrotron emission, Breit—Wheeler pair production, and collective plasma effects can be analysed in a self-consistent way.

Kinetic approaches

- Evolution of particle distribution in classical phase space (see talk by Niel).

- For radiation reaction and Compton scattering: (a) Vlasov equation with modified Vlasov operator, (b) Boltzmann equation with emission rate.

Emission rate

$$\begin{aligned}\frac{d}{dt}f_e &= \int_0^{+\infty} d\gamma_\gamma\, w_\chi(\gamma+\gamma_\gamma,\gamma_\gamma)\, f_e(t,\mathbf{x},\gamma+\gamma_\gamma,\mathbf{\Omega}) \\ &- f_e(t,\mathbf{x},\gamma,\mathbf{\Omega}) \int_0^{+\infty} d\gamma_\gamma w_\chi(\gamma,\gamma_\gamma)\,, \\ \frac{d}{dt}f_\gamma &= \int_1^{+\infty} d\gamma\, w_\chi(\gamma+\gamma_\gamma,\gamma_\gamma)\, f_e(t,\mathbf{x},\gamma+\gamma_\gamma,\mathbf{\Omega}),\end{aligned}$$

- Fokker—Planck form of dynamics.

$$\begin{aligned}\partial_t f_e &+ \nabla\cdot[cu\mathbf{\Omega}f_e] - \frac{1}{mc^2}\partial_\gamma[ecu(\mathbf{\Omega}\cdot\mathbf{E})f_e] \\ &- \frac{e}{p}\nabla_{\mathbf{\Omega}}\cdot[(\mathbb{1}-\mathbf{\Omega}\otimes\mathbf{\Omega})\cdot(\mathbf{E}+u\mathbf{\Omega}\times\mathbf{H})f_e] = \mathcal{C}\,[f_e]\end{aligned}$$

- Gives access to moments. Possible to extend to other processes.

Niel et al., arXiv:1707.02618 (2017)

Kinetic approaches

- Full quantum kinetic approach à la de Groot.
- Based on Wigner function for electron state:

$$\hat{W}_{db}\,(x,p) = \int \frac{d^4y}{(2\pi\hbar)^4}\, e^{-i\frac{p\cdot y}{\hbar}}\, \hat{\Psi}_{db}\left(x+\frac{y}{2}, x-\frac{y}{2}\right)$$

- Slowly varying classical fields:

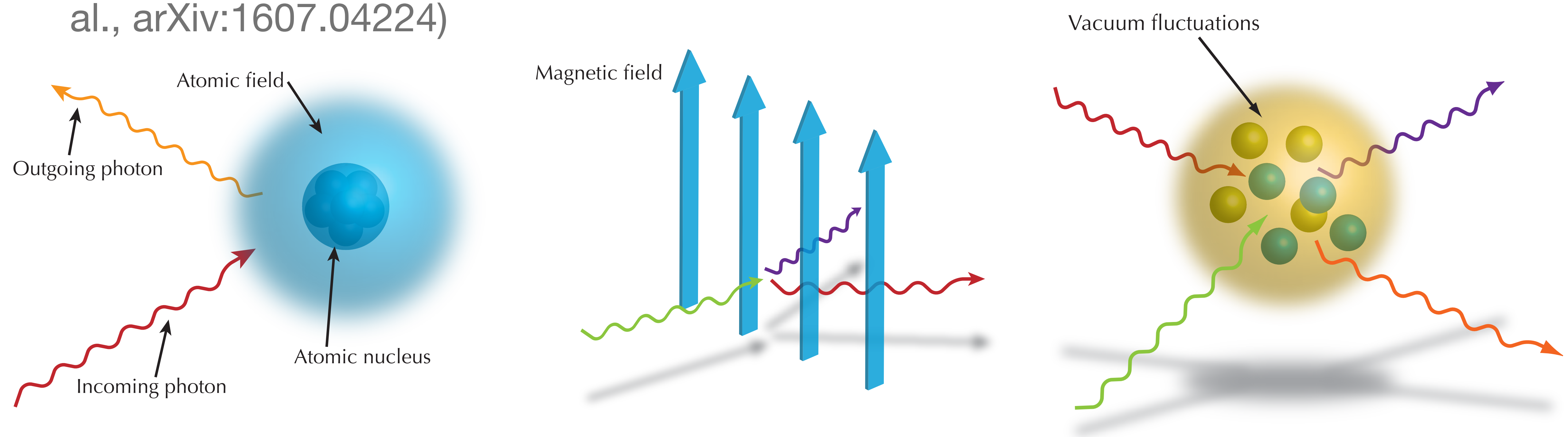
$$\left[m\,(\mathbb{1})_{ca} - (\gamma^\mu)_{ca}\left(p_\mu + \frac{i}{2}\left(\partial^x_\mu - e\,F_{\mu\nu}\,(x)\,\partial^\nu_p\right)\right)\right]\hat{W}_{ab}\,(x,p) = 0$$

- Can in principle be used to obtain closed form dynamical expressions for the processes of interest.
- Can be reduced to a set of molecular dynamics like equations.

de Groot & Suttorp (1972); Ruhl & Herzing, arXiv:1611.03892

The nonlinear quantum vacuum

- Special relativity + Heisenberg's uncertainty relation = virtual pair fluctuations.
- Photons can effectively interact via fluctuating electron-positron pairs.
- Astrophysical applications; laboratory tests of high field QED.
- Implemented numerically (Böhl et al., Phys. Rev. A 92, 032115 (2015); P. Carneiro et al., arXiv:1607.04224)



Marklund & Shukla, Rev. Mod. Phys. **78** (2006); Marklund, Nature Phot. 4 (2010)

The Heisenberg-Euler Lagrangian

- Describes the vacuum fluctuations as an effective field theory, fermionic degrees of freedom integrated out.

$$L = -\frac{\alpha}{2\pi}\epsilon_0 E_{\mathrm{crit}}^2 \int_0^{\mathrm{i}\infty} \frac{dz}{z^3}\mathrm{e}^{-z} \times \left[z^2 \frac{ab}{E_{\mathrm{crit}}^2} \coth\left(\frac{a}{E_{\mathrm{crit}}}z\right) \cot\left(\frac{b}{E_{\mathrm{crit}}}z\right) - \frac{z^2}{3}\frac{(a^2-b^2)}{E_{\mathrm{crit}}^2} - 1\right]$$

$$a = \left[(F^2+G^2)^{1/2} + F\right]^{1/2},\ b = \left[(F^2+G^2)^{1/2} - F\right]^{1/2}$$

$$F \equiv \tfrac{1}{2}(c^2\mathbf{B}^2 - \mathbf{E}^2),\ G \equiv -c\mathbf{E}\cdot\mathbf{B}$$

- Has real and imaginary part. The imaginary part signals "field depletion”, i.e. pair production, the real part defines elastic photon scattering events.

- Can compute equations of motion for test photons in both sub- and super-critical fields.

(Böhl et al., Phys. Rev. A 92, 032115 (2015); P. Carneiro et al., arXiv:1607.04224)

Dispersion relations in external fields

- Result due to HE interactions; nonlinear function of field strengths

$$\omega \approx c|\mathbf{k}|(1 - \tfrac{1}{2}\lambda|\mathbf{Q}|^2) \quad |\mathbf{Q}|^2 \equiv \epsilon_0|\hat{\mathbf{k}} \times \mathbf{E} + c\hat{\mathbf{k}} \times (\hat{\mathbf{k}} \times \mathbf{B})|^2$$

$$\lambda \sim \frac{\alpha}{90\pi}\frac{1}{E_{\mathrm{crit}}}$$

- Result due to HE interactions + lowest order corrections due to finite frequency (still perturbative); nonlinear function of wavenumber

$$\omega \approx c|\mathbf{k}|\left[1 - \tfrac{1}{2}\lambda|\mathbf{Q}|^2\left(1 + 2\sigma\lambda|\mathbf{Q}|^2|\mathbf{k}|^2\right)\right]$$

- Dispersion relation: Hamiltonian for photons. $\dot{x} = \dfrac{\partial\omega}{\partial k}$ $\quad \dot{k} = -\dfrac{\partial\omega}{\partial x}$

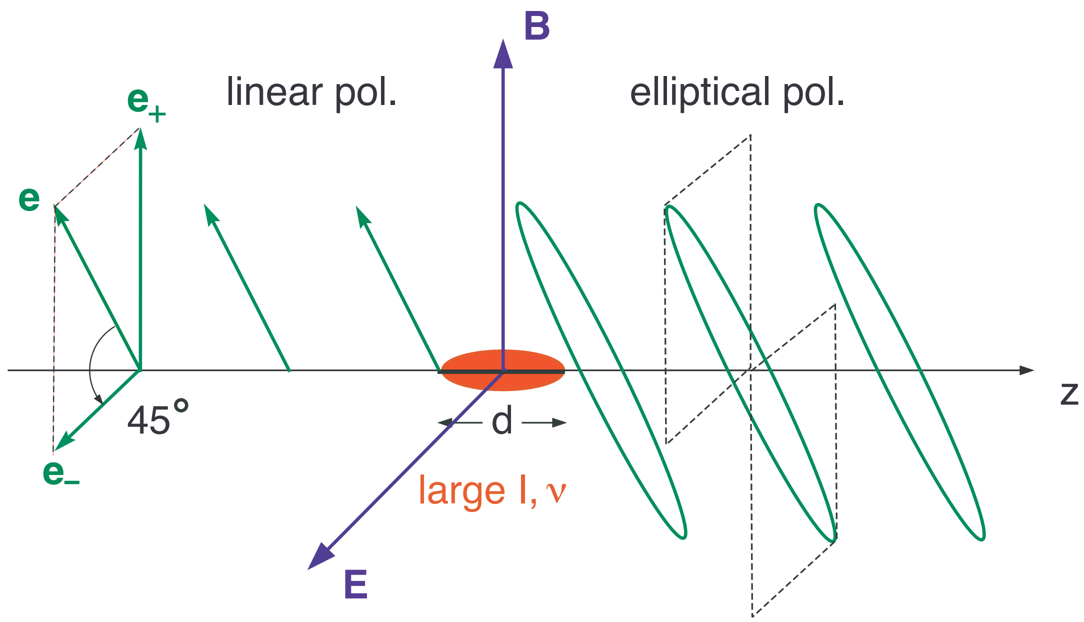
- Phase and group velocites may change in external fields.

Vacuum birefringence

- Anisotropic vacuum, due to e.g. magnetic field (Adler 1970, 1971, Heyl & Hernquist 1997, Dittrich & Gies 1998, Rikken & Rizzo 2000, 2003); permittivity and permeability

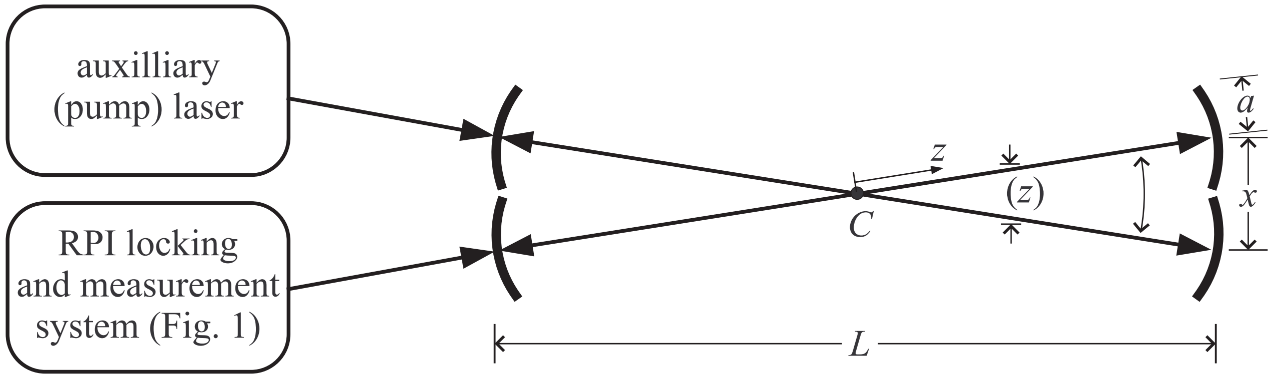
$$\epsilon_{ij} = \delta_{ij} + \frac{4\alpha}{90\pi}\frac{B^2}{E^2_{\text{crit}}}\left(-\delta_{ij} + \frac{7}{2}b_i b_j\right)$$

$$\mu_{ij} = \delta_{ij} + \frac{4\alpha}{90\pi}\frac{B^2}{E^2_{\text{crit}}}\left(\delta_{ij} + 2b_i b_j\right)$$



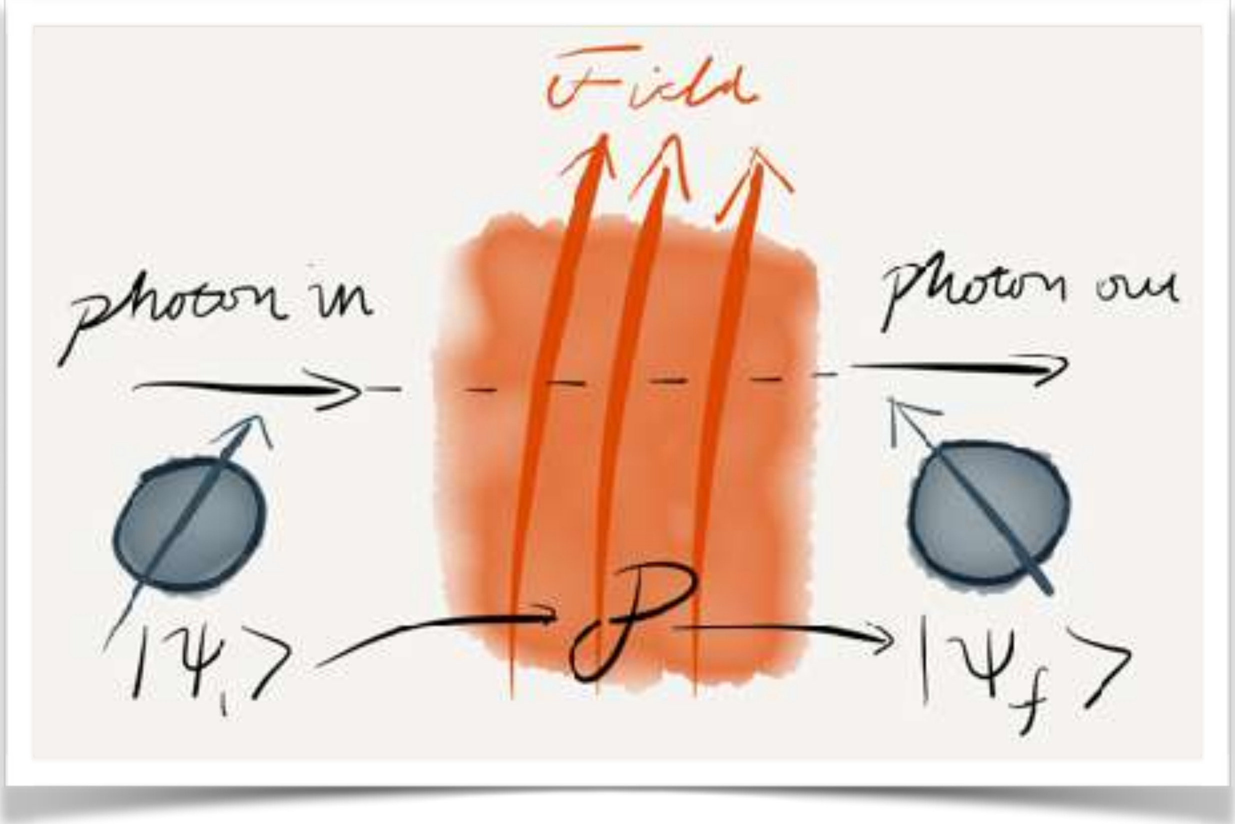
- Refractive index different for different propagation angles, relative external field

$$n - 1 \sim \frac{\alpha}{90\pi}\frac{B^2}{E^2_{\text{crit}}}\sin^2\theta$$



Vacuum birefringence

- Instead of effective field theory approach, what is happening at the photon level?
  - Start with full QED and calculate the helicity flip probability,
  - gives back the effective field theory approach when summing up the photons,
  - use this to check the properties of birefringence in different field configurations, structured pulses etc.,
  - shows that birefringence is robust towards field structure,
  - can also be used with few-photon sources.



V. Dinu et al., Phys. Rev. D 89, 125003 (2014); Phys. Rev. D 90, 045025 (2014)

Summary

Particle simulation of plasmas

John M. Dawson

Department of Physics, University of California, Los Angeles, California 90024

”Proper treatment of systems where both the microscopic and macroscopic behavior are important will undoubtedly challenge simulation physicists for many years to come”

Energy and intensity

4. QED plasmas
- Many-body physics in strong fields

3. Relativistic plasmas
- Fully relativistic plasmas
- Fully quantum mechanical single particle processes

2. Semiclassical plasmas
- Collisional processes.
- Ionization.

1. Classical plasmas
- MHD
- Gyrokinetics/Fokker-Planck
- Particle-in-cell

Open questions

A. Some common platform for code development!

B. Make our codes more complete: including further processes with higher accuracy.

C. Overcome multiple scale issues (volume, timescale…) for proper experimental analysis.

D. Accurate interplay with experiments (e.g., detailed data input).

E. The breakdown of the locally constant crossed field approximation?

F. The role of coherent multi-photon effects.

G. Depletion mechanisms of background fields; the breakdown of the background field approximation?

H. The transition between the *S*-matrix approach and equations of motion. No trajectories in QED.

I. Transition times in quantum processes from in and out states? Compare ionization.

J. Non-equilibrium many-body QFT approach. Compare condensed matter, transport theory of solids, and TDFT development.

Example: event generators

$$\frac{dW_{\rm rad}(\varepsilon_\gamma)}{d\varepsilon_\gamma} = -\frac{\alpha m^2 c^4}{\hbar \varepsilon_e^2}\left\{\int_x^\infty \mathrm{Ai}(\xi)d\xi + \left(\frac{2}{x} + \chi_\gamma\sqrt{x}\right)\mathrm{Ai}'(x)\right\}$$

$$\chi = \frac{e\hbar}{m^3c^4}\sqrt{\left(\frac{\varepsilon \boldsymbol{E}}{c} + \boldsymbol{p}\times\boldsymbol{H}\right)^2 - (\boldsymbol{p}\cdot\boldsymbol{E})^2}$$

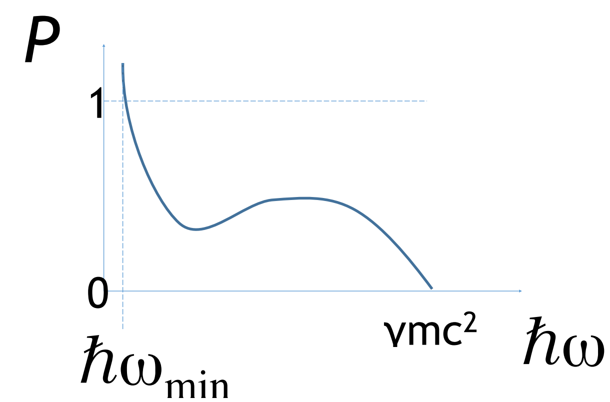
$$x = (\chi_\gamma/\chi_e\chi_e')^{2/3}$$

Inverse sampling method

E.N. Nerush et al. PRL **106**, 035001 (2011)

N.V. Elkina et al. PR STAB **14**, 054401 (2011)
(+Alternative Event Generator)

C.P. Ridgers et al. J. Comput. Phys. **260**, 273 (2014)

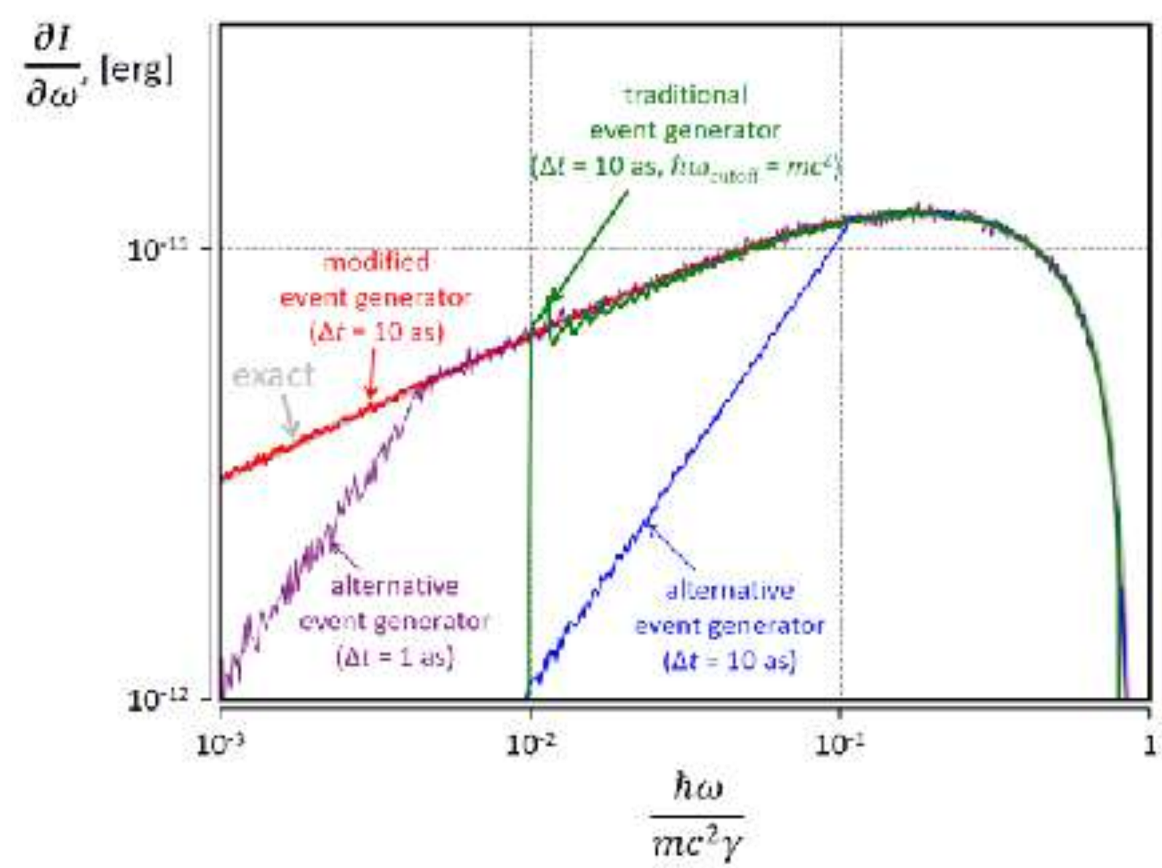


Problems:
- infrared cutoff
- combining with classical RR

Modified Event Generator

A. Gonoskov et al., PRE **92** (2), 023305 (2015)

- no infrared cutoff
- describes also classical RR
- close to minimal requirements to the time step



Thinning and merging

Thinning

G. Lapenta, JCP(1994);
E. Nerush et al., PRL (2011);
A. Timokhin (2010)
A. Gonoskov et al., PRE (2015)

Merging

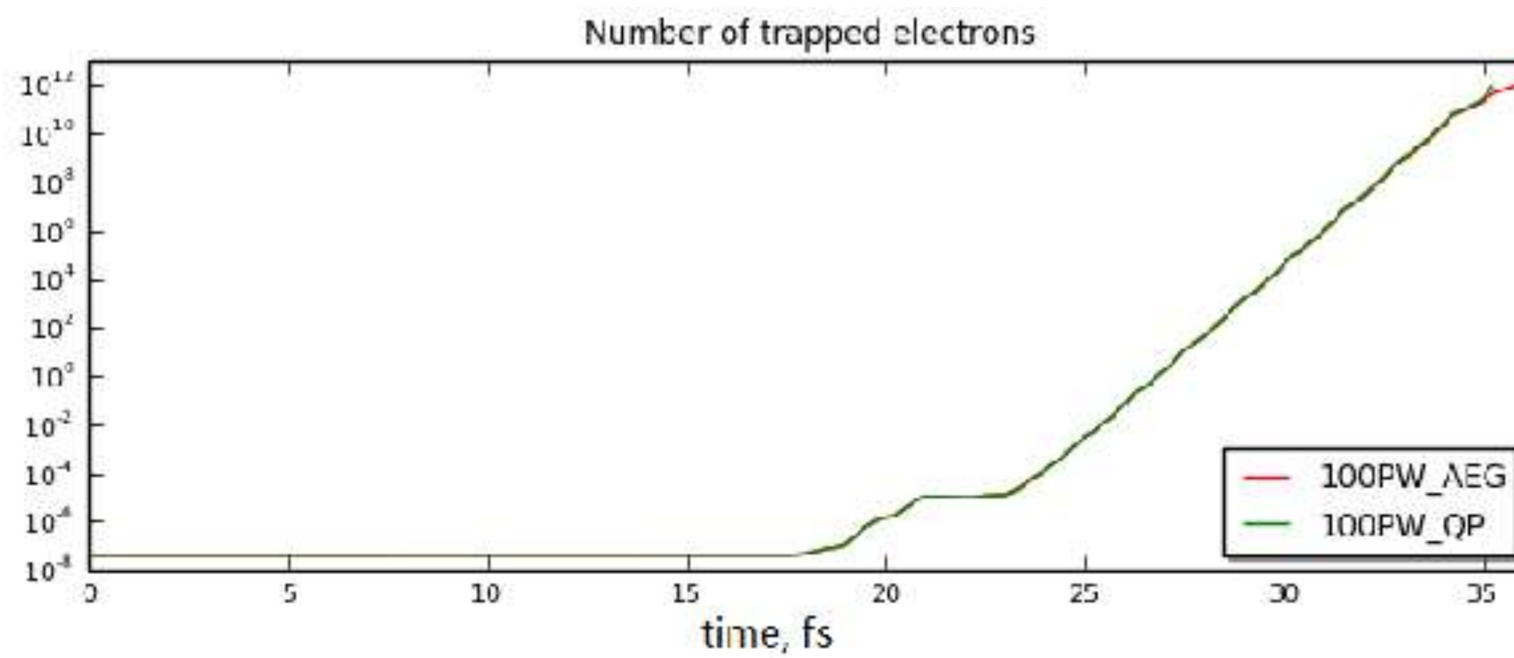
G. Lapenta, JCP (2002)

Merging with preservation of conservation laws

M. Vranic et al., CPC (2015)

Thinning with exact preservation of distribution functions and conservation laws

A. Gonoskov, arXiv:1607.03755 (2016)

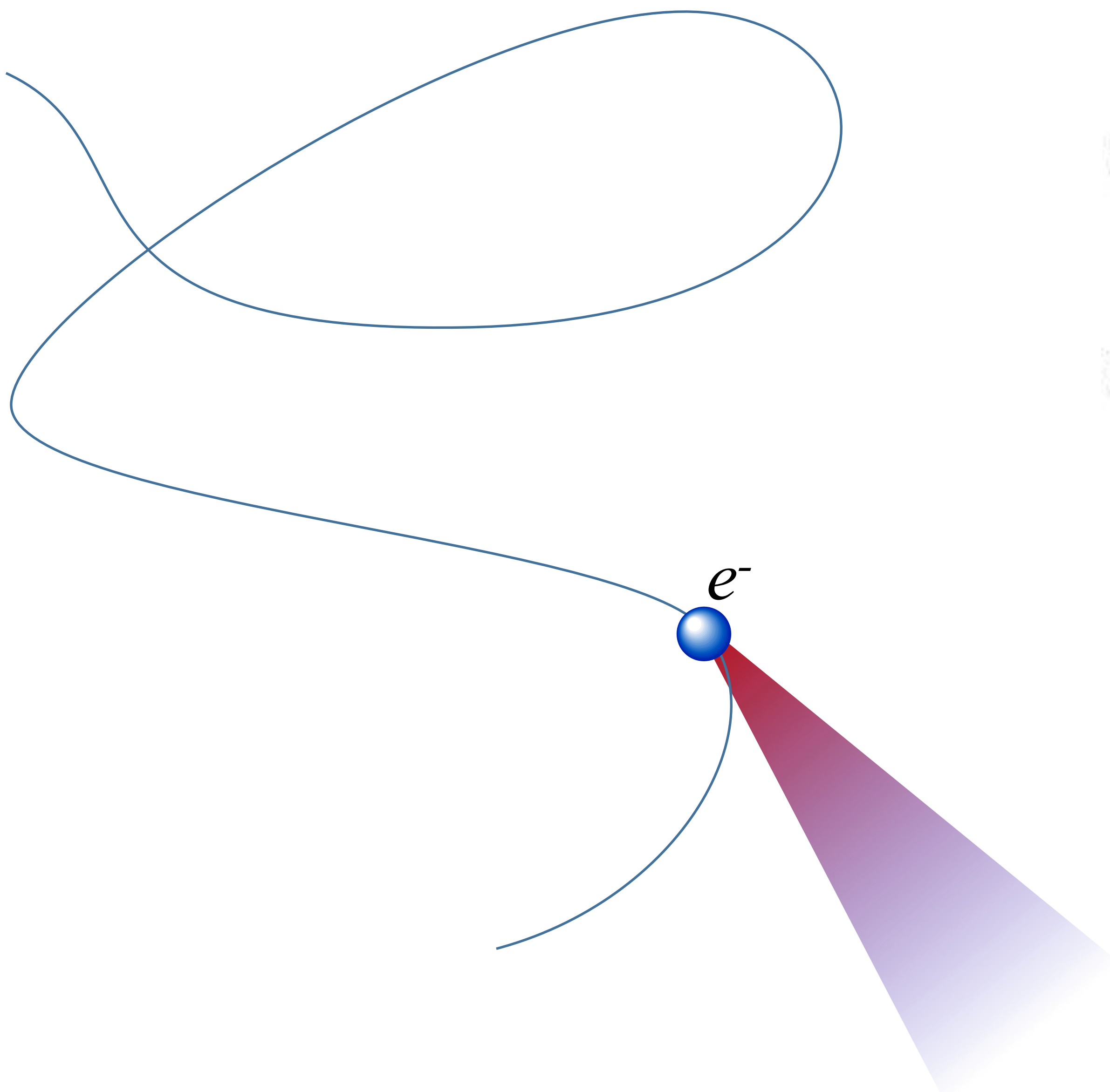


Introduction

$$I_{\text{synch}} = \frac{2}{3}\frac{e^2 m^2 c^3}{\hbar^2}\chi^2,$$

$$\chi = \gamma\frac{H_{\text{eff}}}{E_S}, \quad E_S = \frac{m^2c^3}{e\hbar} \simeq 10^{18}\frac{\text{V}}{\text{m}}, \quad H_{\text{eff}} = \frac{1}{c}\frac{\partial \mathbf{p}}{\partial t}\Big|_{\perp}.$$

$$\chi = \frac{2}{3}\frac{\hbar\omega_c}{mc^2\gamma}, \quad \text{for } \chi \sim 1, \quad I^{\text{q}}_{\text{synch}} \approx 0.37\frac{e^2m^2c^3}{\hbar^2}\chi^{2/3}.$$



Effects:

- radiation friction $I > 10^{23}$ W/cm^2
- discontinuity $I > 10^{24}$ W/cm^2
- cascades $I > 10^{25}$ W/cm^2

Model for classical RR

Extensive literature on this topic, as well as many different implementations, see, e.g.,

Di Piazza, Lett. Math. Phys. 83, 305 (2008)

Bell & Kirk, PRL 101, 200403 (2008)

Di Piazza et al., PRL 105, 20403 (2010)

Bulanov et al., PoP 17, 063102 (2010)

Sokolov et al., PoP 18, 093109 (2011)

Thomas et al., PRX 2, 041004 (2012)

Di Piazza et al., RMP 84, 1177 (2012)

Schlegel & Tikhonchuk, NJP 14, 073034 (2012)

Chen et al., PRSTAB 16, 030701 (2013)

Mackenroth et al., PPCF 55, 124018 (2013)

M. Vranic et al. Comput. Phys. Commun. 204, 141-151 (2016)

Ilderton & Torgrimsson, PRD 88, 025021 (2013)

A. Ilderton & G. Torgrimsson, PRD 88, 025021 (2013)

Ridgers et al., J. Comp. Phys. 260, 273 (2014)

Yo

Ta

etc

Radiation Reaction	$\mathcal{O}(e^2)$	$\mathcal{O}(e^4)$
Abraham Lorentz Dirac (LAD)	✓	?
Landau Lifshitz (LL)	✓	?
Eliezer Ford O'Connell (EFO)	✓	?

Including classical radiation reaction in PIC codes

- PIC schemes work with super particles where the charge to mass ratio is kept fixed for each species q/m.
- This works because the acceleration due to the Lorentz force depends on q/m.
- Not true for RR force. However

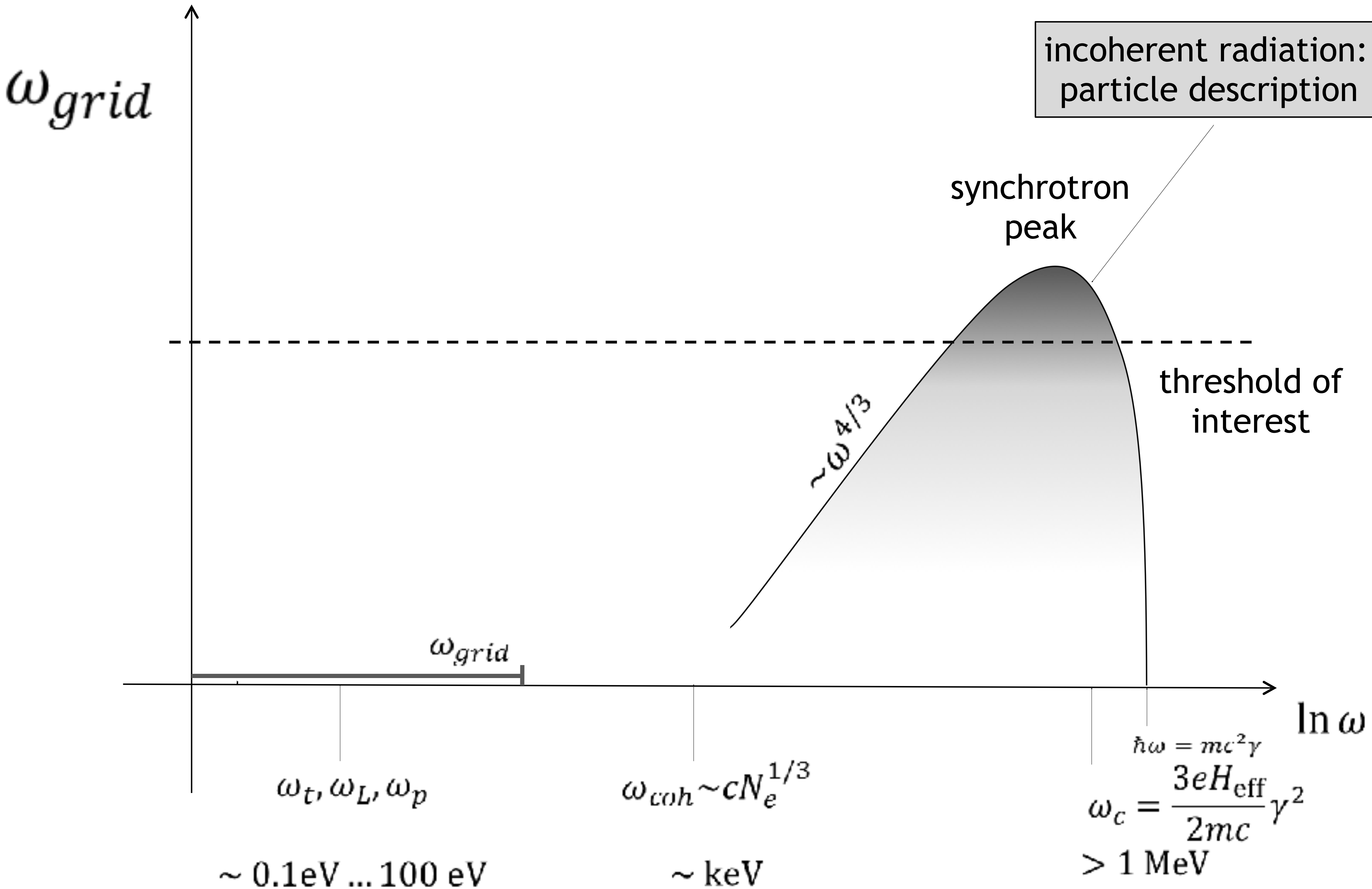
Let us consider a macro particle that represents η electrons. The charge of the macro particle is $e_m = \eta e$, and the mass is $m_m = \eta m_e$. For a single particle with the same mass and charge as the macro particle, the radiation reaction would be η times stronger than in the case of a single electron:

$$\frac{F_{RR}}{F_L} \propto \frac{(\eta e)^3}{(\eta m_e)^2} = \eta \frac{e^3}{m_e^2} \tag{12}$$

and the trajectory of such particle would be different than the trajectory of a single electron (Fig. 1). This result would be equivalent to assuming that η electrons are radiating coherently. As a consequence, the results of a PIC simulation would be qualitatively different for different number of particles per cell or different cell sizes. To obtain the correct dynamics of a macro-particle, it is therefore essential to use the real charge and mass to calculate the correct radiation reaction coefficient for a particular particle species. This approach yields the same result regardless of the macro-particle weight.

M. Vranic et al. Comput. Phys. Commun. 204, 141-151 (2016)

Avoiding double-counting of RR: Dual treatment of the EM field

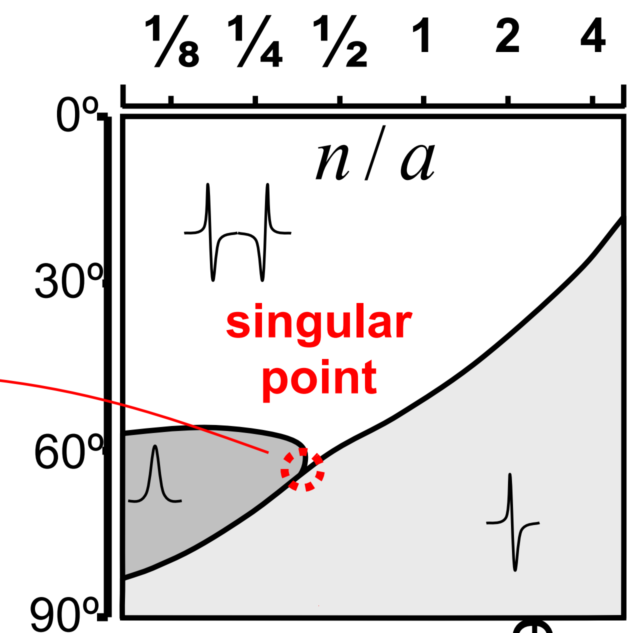


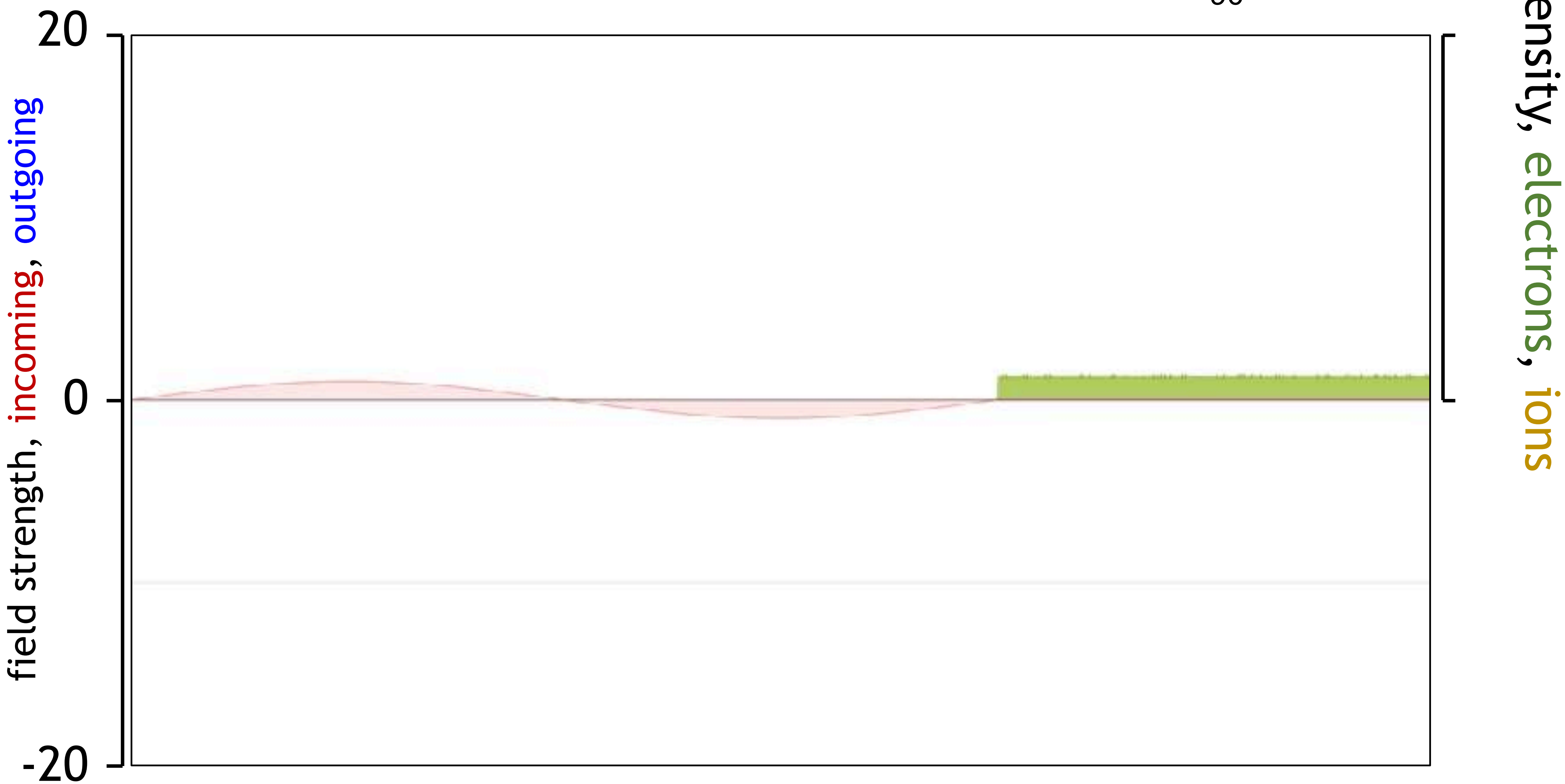
A. Gonoskov et al., PRE 92 (2), 023305 (2015)

Dual treatment of the EM field: validation with the worst case scenario

Relativistic electronic spring (RES) theory*:

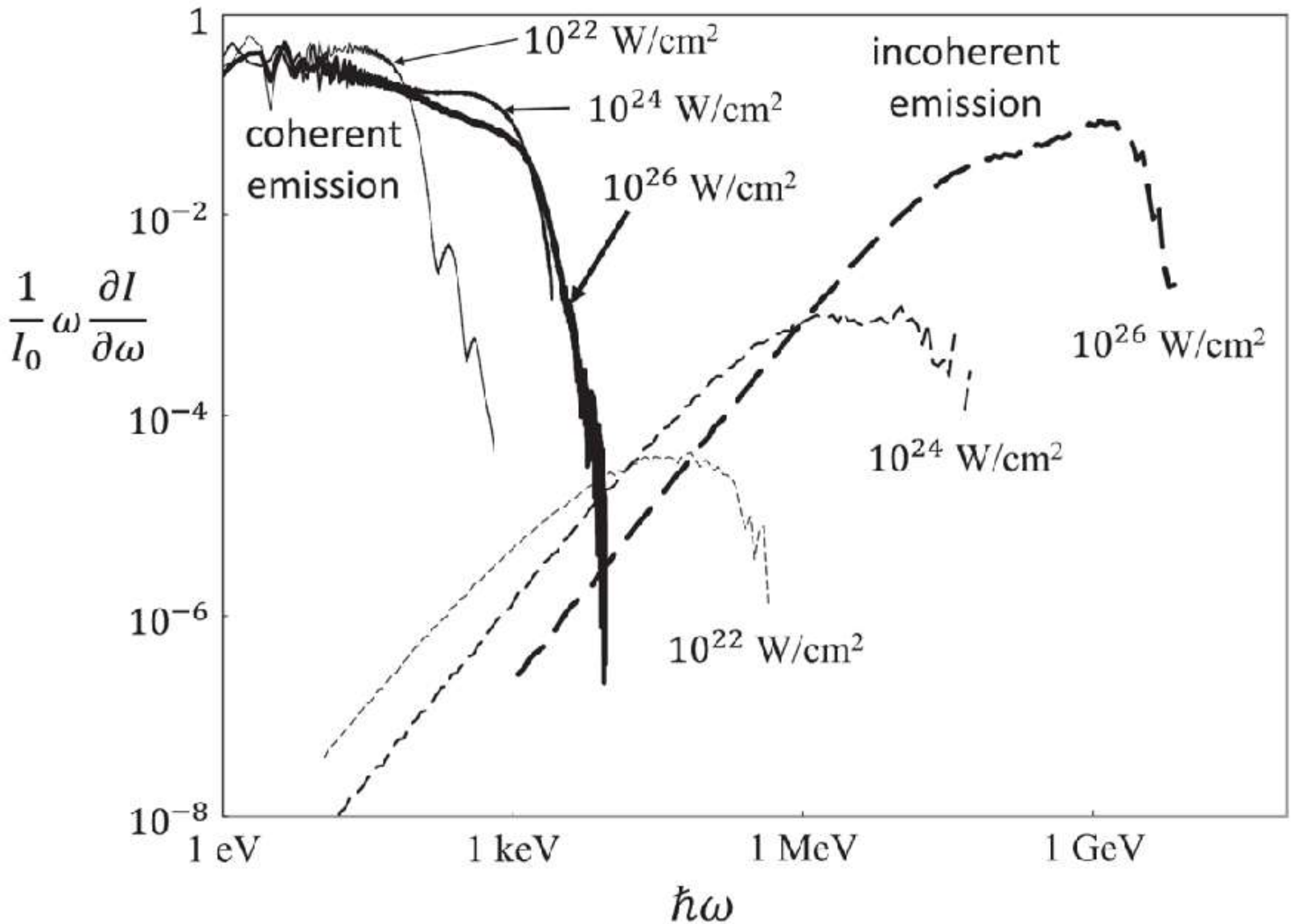
The plasma acts as a slingshot, accumulating and releasing the energy of an optical cycle in an XUV burst both 100 times shorter in duration and 100 times higher in intensity.





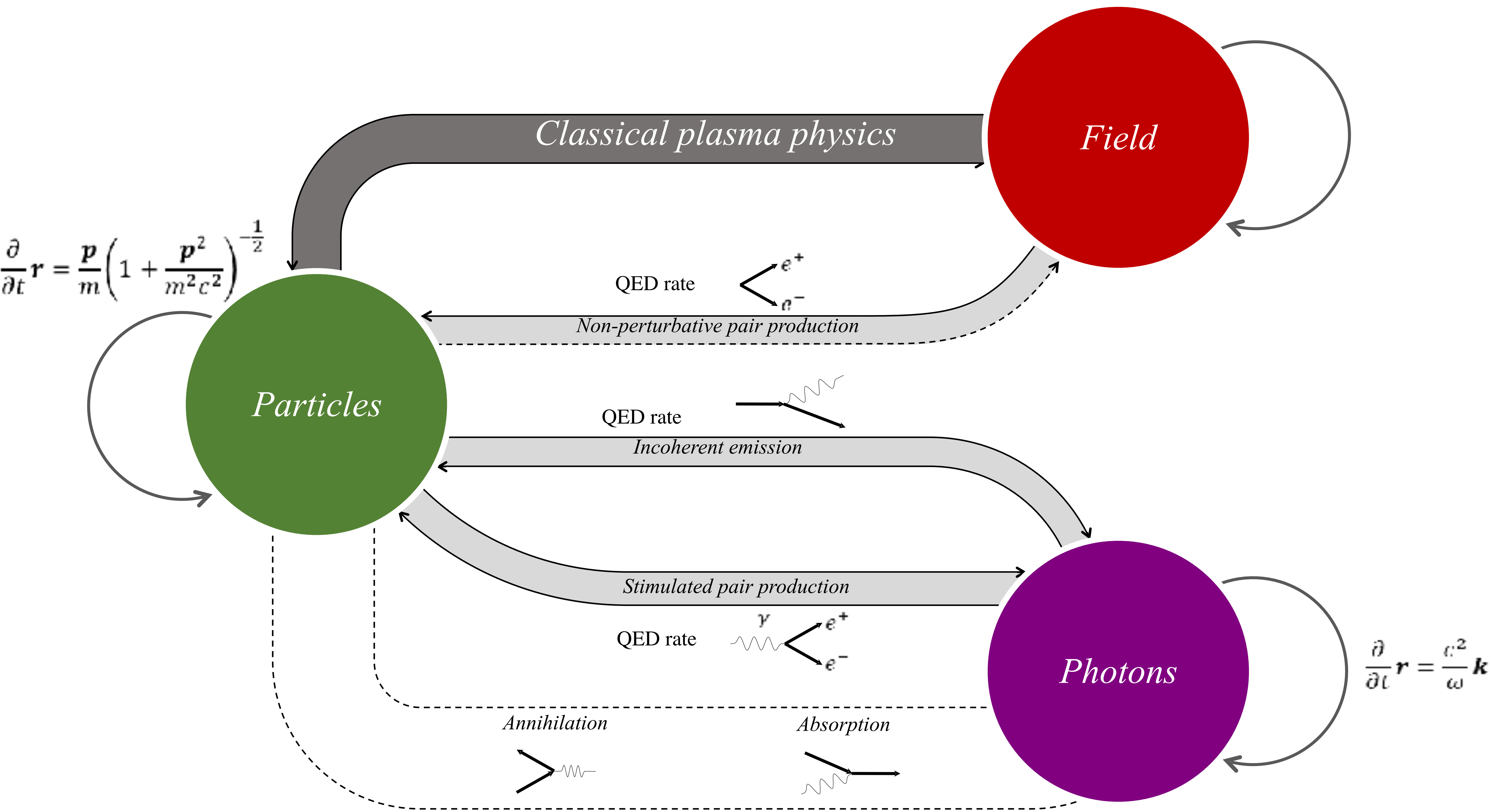
* A. Gonoskov et al., PRE 84, 046403 (2011); PhD thesis (2014)

Dual treatment of the EM field: validation with the worst case scenario



A. Gonoskov et al., PRE 92 (2), 023305 (2015)

Dual treatment of the electromagnetic field



Quantum radiation reaction

Overestimation of radiation losses

$$\boxed{\chi = \frac{2}{3}\frac{\hbar\omega_s}{mc^2\gamma} = \gamma\frac{H_{\text{eff}}}{E_S}}$$

$$I^{cl} = \frac{2}{3}\frac{e^2m^2c^3}{\hbar^2}\chi^2$$

$$I^q \approx 0.37\frac{e^2m^2c^3}{\hbar^2}\chi^{2/3}$$

Sokolov et al., PoP 18, 093109 (2011)
J L Martins et al. PPCF **58** (2016)

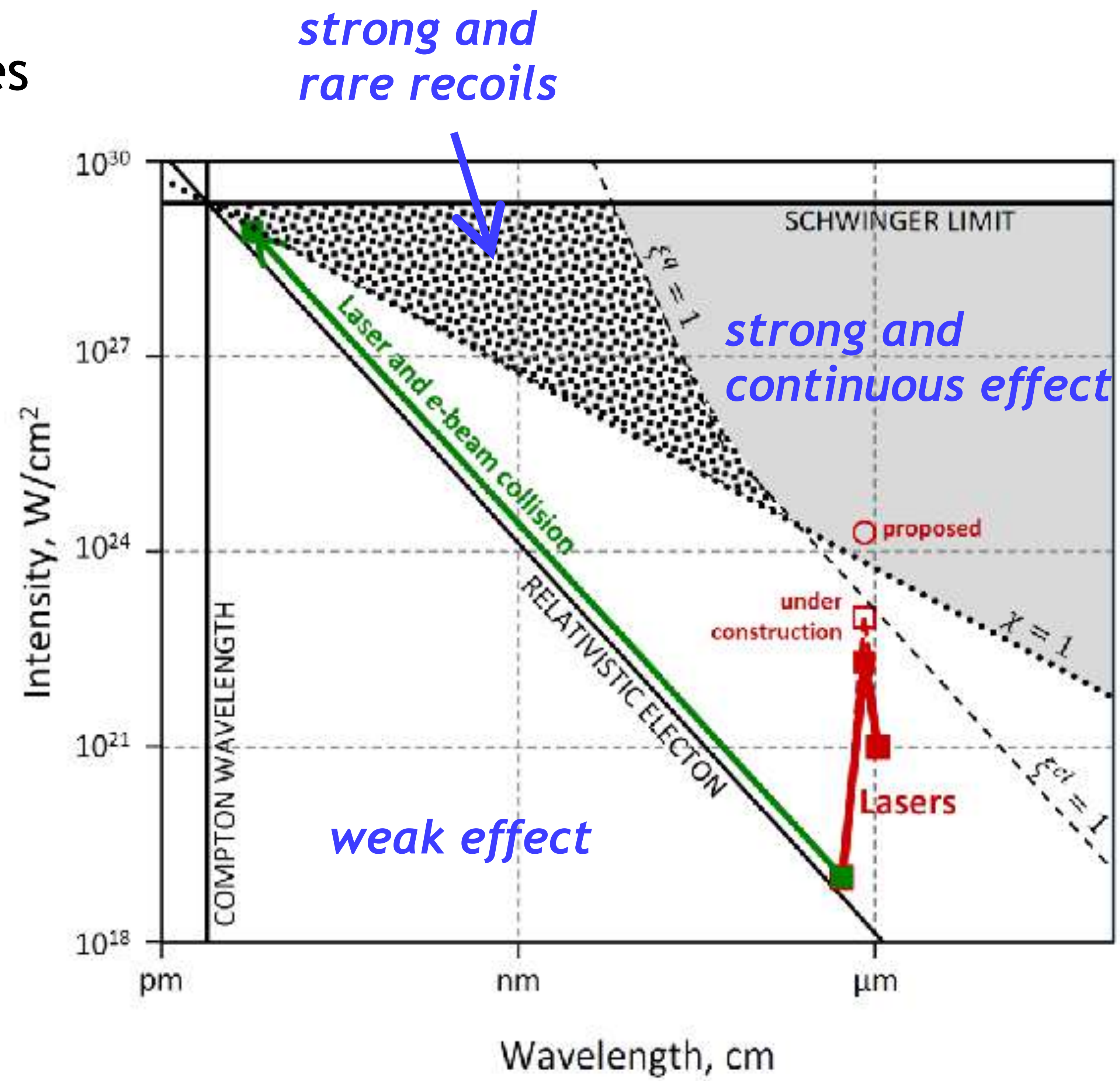
Discreetness of emission

$$\boxed{\xi = 2\pi\frac{\tau_t}{T_t}}$$

$v<<c$: $\xi \approx 200\frac{c^2}{v^2}$

$v \sim c$: $\xi^{cl} \approx 300a^{-1}$

$$\xi^q \approx 200\left(\frac{\lambda}{\lambda_C}a\right)^{-1/3}$$



A. Gonoskov et al., PRE 92 (2), 023305 (2015)

Probabilistic treatment of quantum RR

$$\frac{dW_{\mathrm{rad}}(\varepsilon_\gamma)}{d\varepsilon_\gamma} = -\frac{\alpha m^2 c^4}{\hbar \varepsilon_e^2}\left\{\int_x^\infty \mathrm{Ai}(\xi)d\xi + \left(\frac{2}{x} + \chi_\gamma \sqrt{x}\right)\mathrm{Ai}'(x)\right\}$$

$$\chi = \frac{e\hbar}{m^3 c^4}\sqrt{\left(\frac{\varepsilon \boldsymbol{E}}{c} + \boldsymbol{p}\times\boldsymbol{H}\right)^2 - (\boldsymbol{p}\cdot\boldsymbol{E})^2}$$

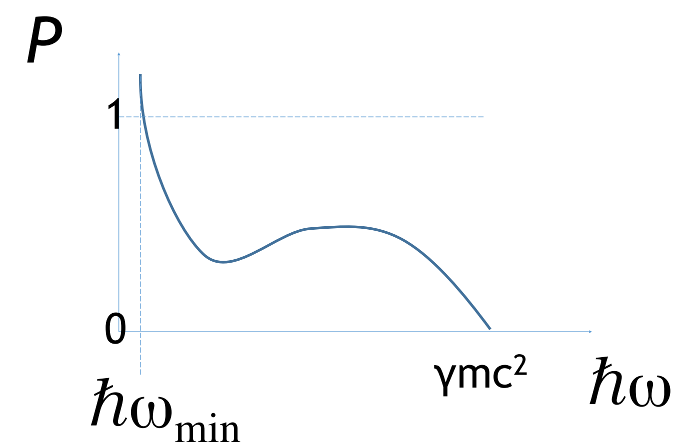
$$x = (\chi_\gamma / \chi_e \chi_e')^{2/3}$$

Inverse sampling method

E.N. Nerush et al. PRL **106**, 035001 (2011)

N.V. Elkina et al. PR STAB **14**, 054401 (2011) (+Alternative Event Generator)

C.P. Ridgers et al. J. Comput. Phys. **260**, 273 (2014)

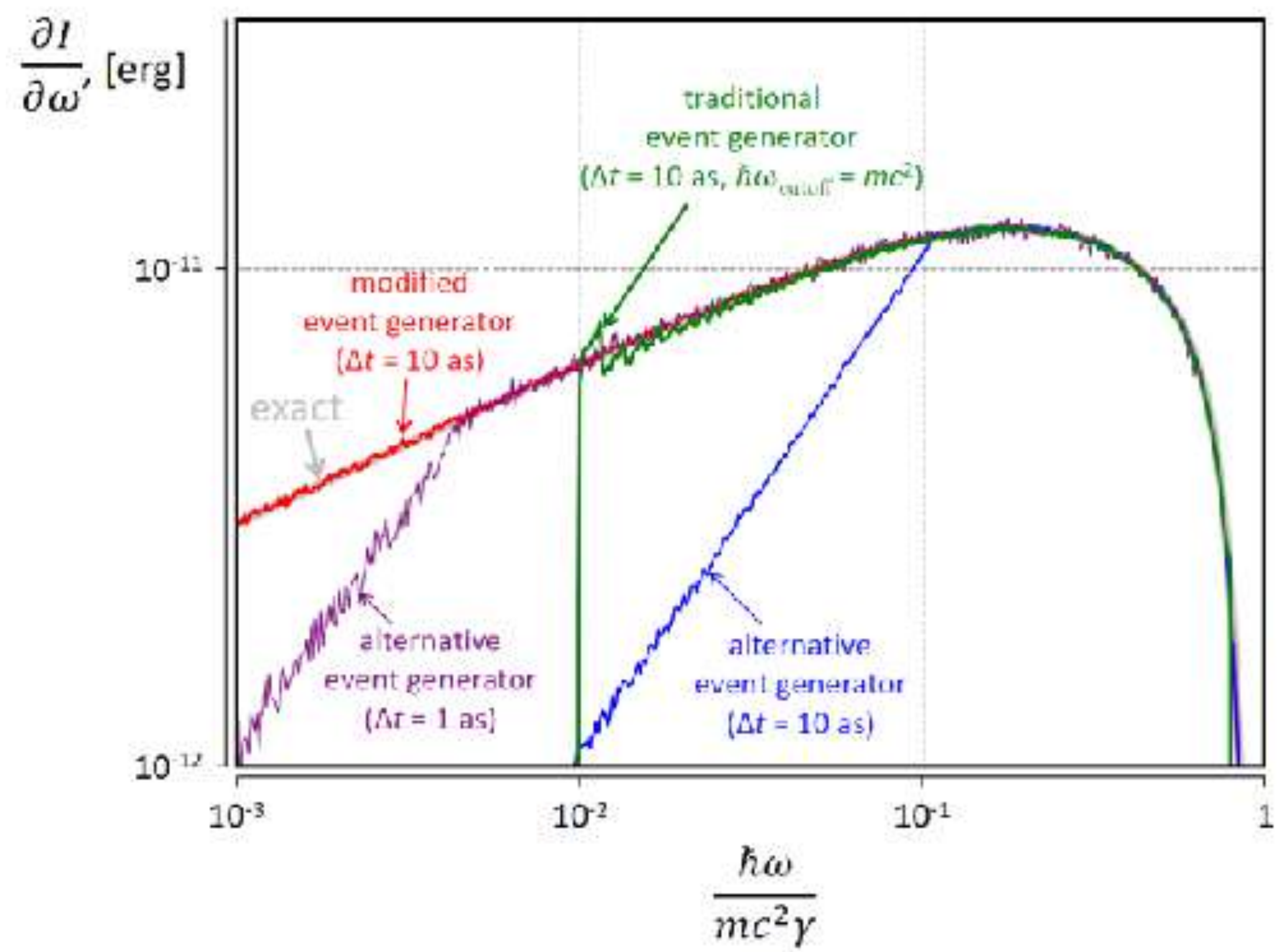


Problems:
- infrared cutoff
- combining with classical RR

Modified Event Generator

A. Gonoskov et al., PRE **92** (2), 023305 (2015)

- no infrared cutoff
- describes also classical RR
- close to minimal requirements to the time step



Unbinding requirements on the time step

Rate:

$$P_{\mathrm{p}}(\delta_e) = \left[\Delta t \frac{e^2 mc}{\hbar^2}\right] \frac{\sqrt{3}}{2\pi} \frac{\chi_\gamma}{\hbar\omega/(mc^2)} (\delta_e - 1)\,\delta_e \left\{F_1(z_{\mathrm{p}}) - \frac{3}{2}\chi_\gamma z_{\mathrm{p}} F_2(z_{\mathrm{p}})\right\}$$

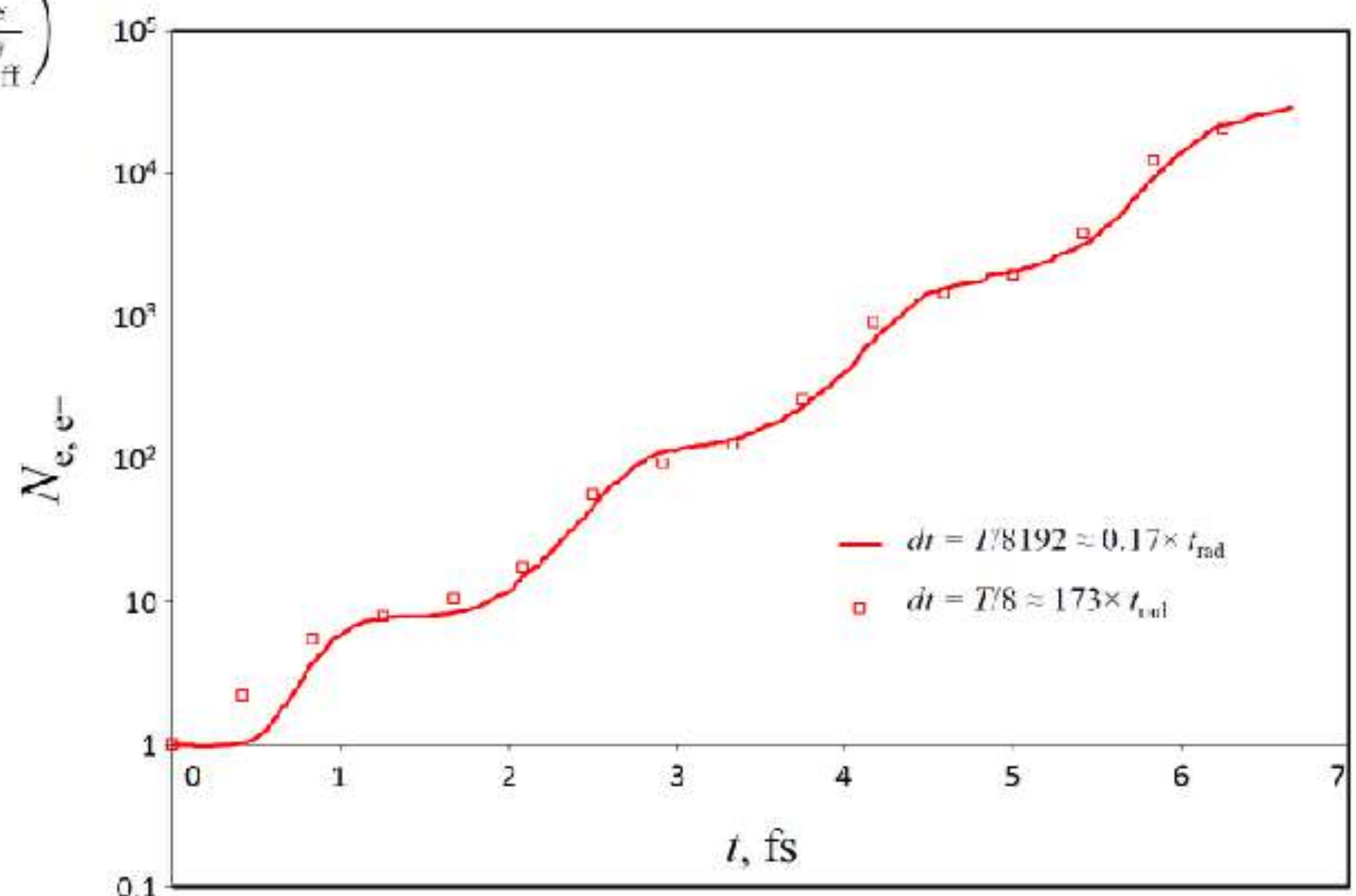
$$z_{\mathrm{p}} = \frac{2}{3}\frac{1}{\chi_\gamma (1-\delta_e)\,\delta_e}$$

Requirements on time step (modified event generator):

Emission: $\Delta t \ll \frac{137}{2\pi}\frac{\lambda_C}{c}\min\left\{0.67\gamma^{1/3}\left(\frac{E_s}{H_{\mathrm{eff}}}\right)^{2/3}, 2\left(\frac{E_s}{H_{\mathrm{eff}}}\right)\right\}$

Pair production: $\Delta t \ll 130\frac{\lambda_C}{c}\left(\frac{E_s}{H_{\mathrm{eff}}^{\gamma}}\right)$

$$\Delta t \ll 14\frac{\lambda_C}{c}\left(\frac{E_s}{H_{\mathrm{eff}}}\right)^{2/3}$$



Adaptive event generator

- Modified Event Generator
- Timestep subcycling
- Local cascade resampling

A. Gonoskov et al., PRE 92 (2), 023305 (2015)

PICADOR code

3D EM-PIC code / open for collaborations

Project leaders
Arkady Gonoskov, Chalmers (Sweden), Iosif Meyerov, UNN (Russia), Evgeny Efimenko, IAP RAS (Russia)

Core developers (UNN, Russia)
Sergey Bastrakov, Igor Surmin, Anton Larin, Anatoly Rozanov

Collaborators, extension developers, users
Alexey Bashinov, Artem Korzhimanov, Alexander Muraviev IAP RAS (Russia); Joel Magnusson, Benjamin Svedung Wettervik, Chalmers; Erik Wallin, Jens Zamanian, Umea Univ. (Sweden); Felix Mackenroth, MPIPCS (Germany); Manuel Blanco (Spain).

Numerical schemes: FDTD, NDF, Boris, CIC, TSC, Esirkepov, Villasenor-Buneman, filters

Optimization: Adaptive load balancing, vectorization, supercells

MPI + OpenMP / GPU / XeonPhi + heterogeneou

92%(2k) PPC, ns = 7.9(E5)/ 63(core) 14 / 3.3 (KNL7250)

ELMIS 3D spectral EM-PIC code
num. dispersion free + Poisson's eq. solver + spectral filtering
MPI (70%1k), PPC = 300ns(core)

Module Development Kit (MDK)

- scripting language for input/output and modules
- controls hierarchy (MPI+OpenMP/XeonPhi)
- handles and optimizes data transferring/collecting
- protects and supports development
- assists post-processing

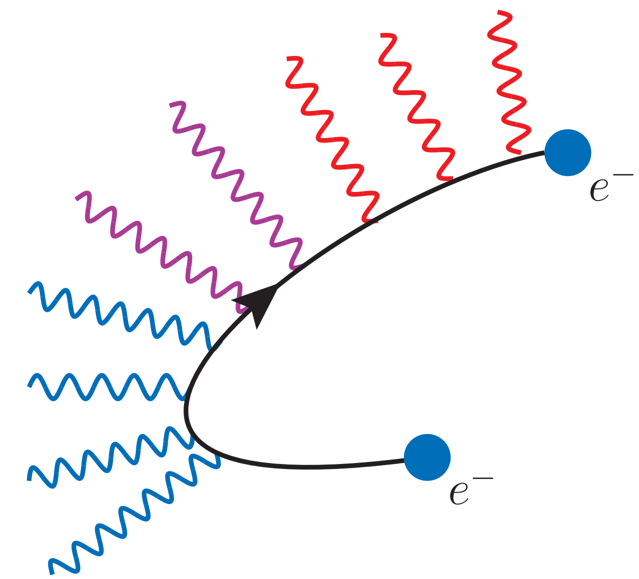
- QED (A. Gonoskov et al., PRE (2015)
- Resampling
- Ionization
- RR (LL)
- Diagnostic tools
- On-the-fly 1D/2D/3D graphics

PICADOR

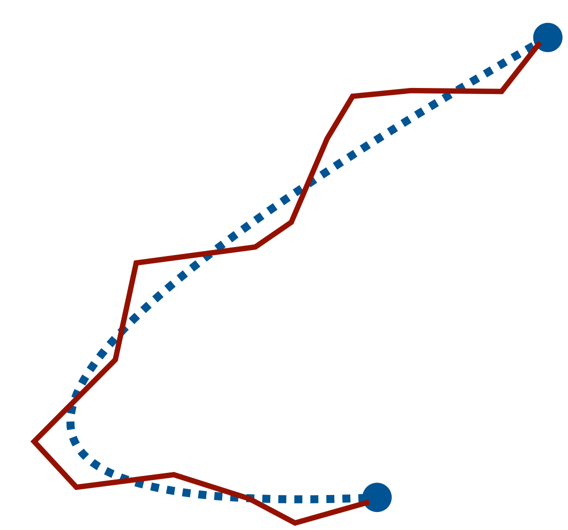
Open questions

- The role of coherent multi-photon effects.
- Depletion mechanisms of background fields; the breakdown of the background field approximation?
- The transition between the *S*-matrix approach and equations of motion: meaningful? No trajectories in QED.
- Transition times in quantum processes from in and out states? Compare ionization.
- Many-body relativistic quantum processes in strong fields: possible? Path integral approach?

Introduction



- Accelerated charged particles emit radiation.
- Strong enough accelerating field: emitted radiation will give significant momentum kick to electron.
- Momentum kick: radiation reaction.*
- Classical regime: continuous emission, Landau-Lifshitz description.
- Quantum regime: discrete, stochastic emission.
- Straggling‡: stochastic emission makes electron reach classically forbidden regions of laser pulse.



*Di Piazza et al., Rev. Mod. Phys. **84**, 1177 (2012)
‡Shen & White, Phys. Rev. Lett. **28**, 455 (1972)
‡Blackburn et al., Phys. Rev. Lett. **112**, 015001 (2014)

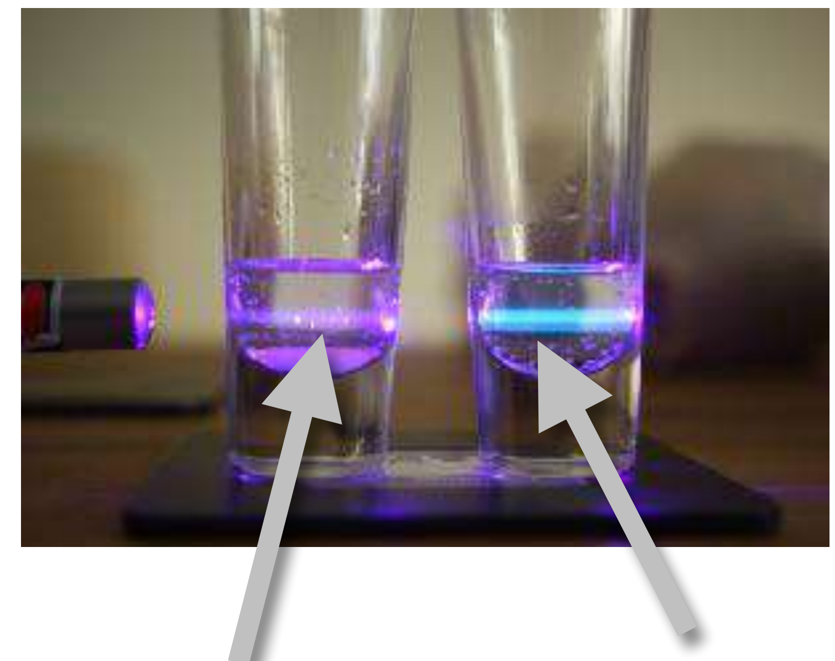
Quenching

- Fluorescence quenching through Dexter electron transfer.*

- Excited electrons transfer between molecules on a non-radiative path of length ℓ.

- Decreases fluorescence, purely quantum.

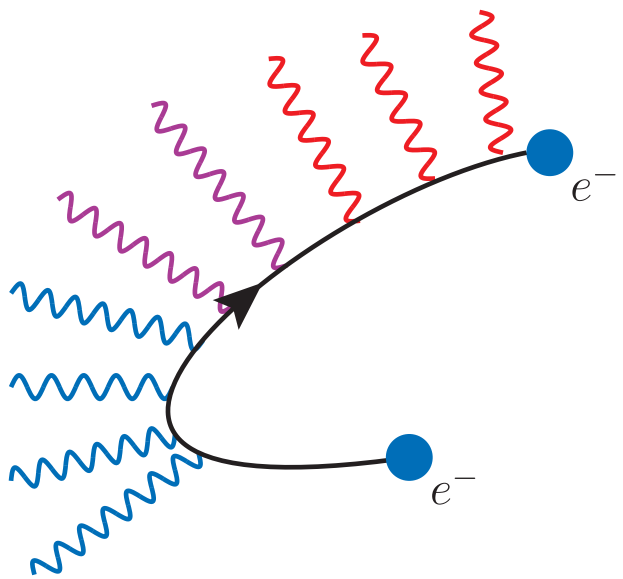
- Exponentially suppressed by path length ℓ: $\sim \exp(-\ell/L)$



* Dexter, J. Chem. Phys. **21**, 836 (1953)

Idea

- Probability of one emitted photon: $P_1(\tau, I)$ where τ is the pulse length and I is the pulse intensity.
- Growing function of τ and I.
- Probability of zero emitted photons:* $P_0 = \exp(-P_1)$
- Exponentially suppressed by pulse length.
- Motion without emission $\leftrightarrow$ motion according to Lorentz force, non-radiative.
- We take steps of increasing complexity.



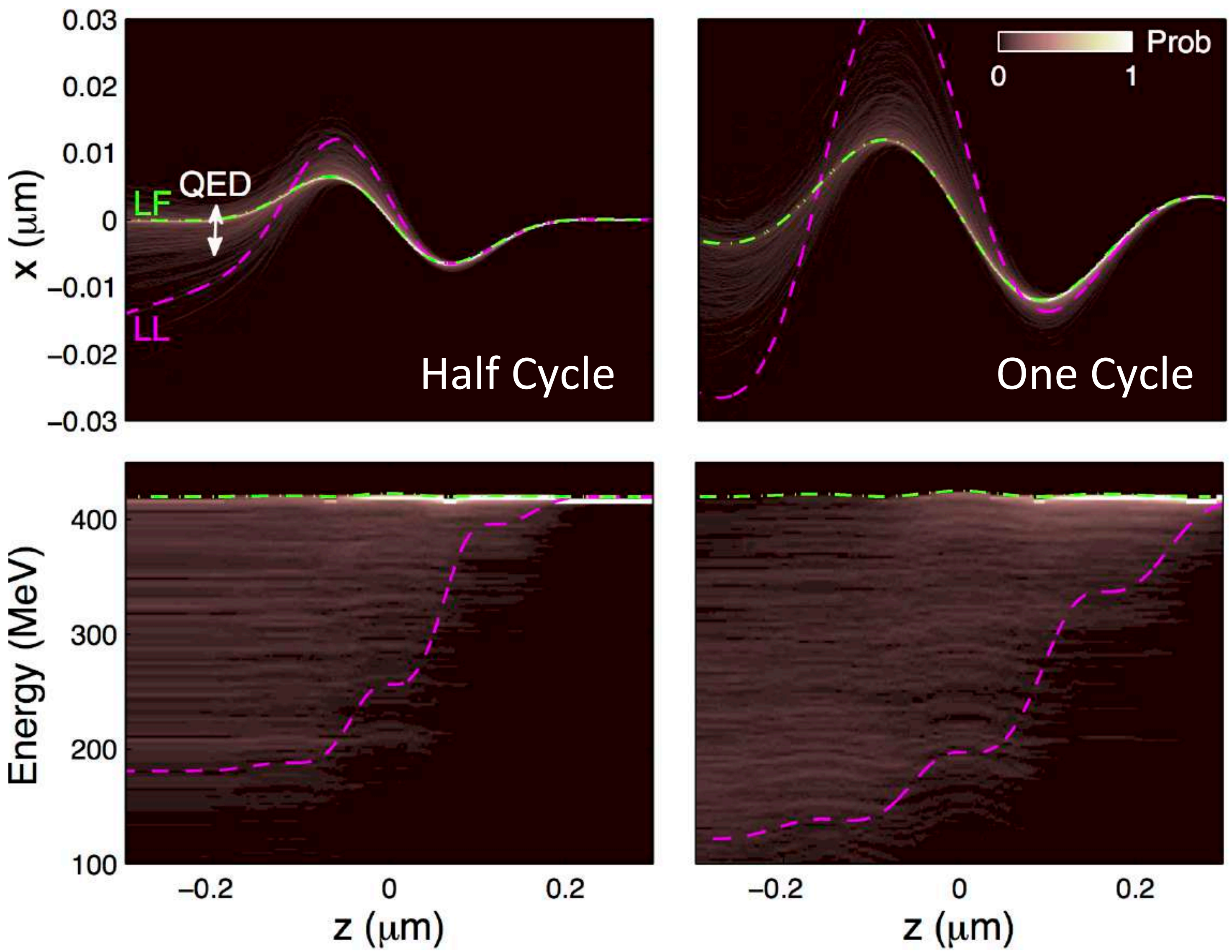
We therefore look at short pulses!

*Yennie et al., Ann. Phys. (N.Y.) **13**, 379 (1961)
*Ritus, J. Sov. Laser Res. **6**, 497 (1985)

1D electron motion

- Motion without emission $\leftrightarrow$ motion according to Lorentz force. Single electron motion.

Direction of electron motion

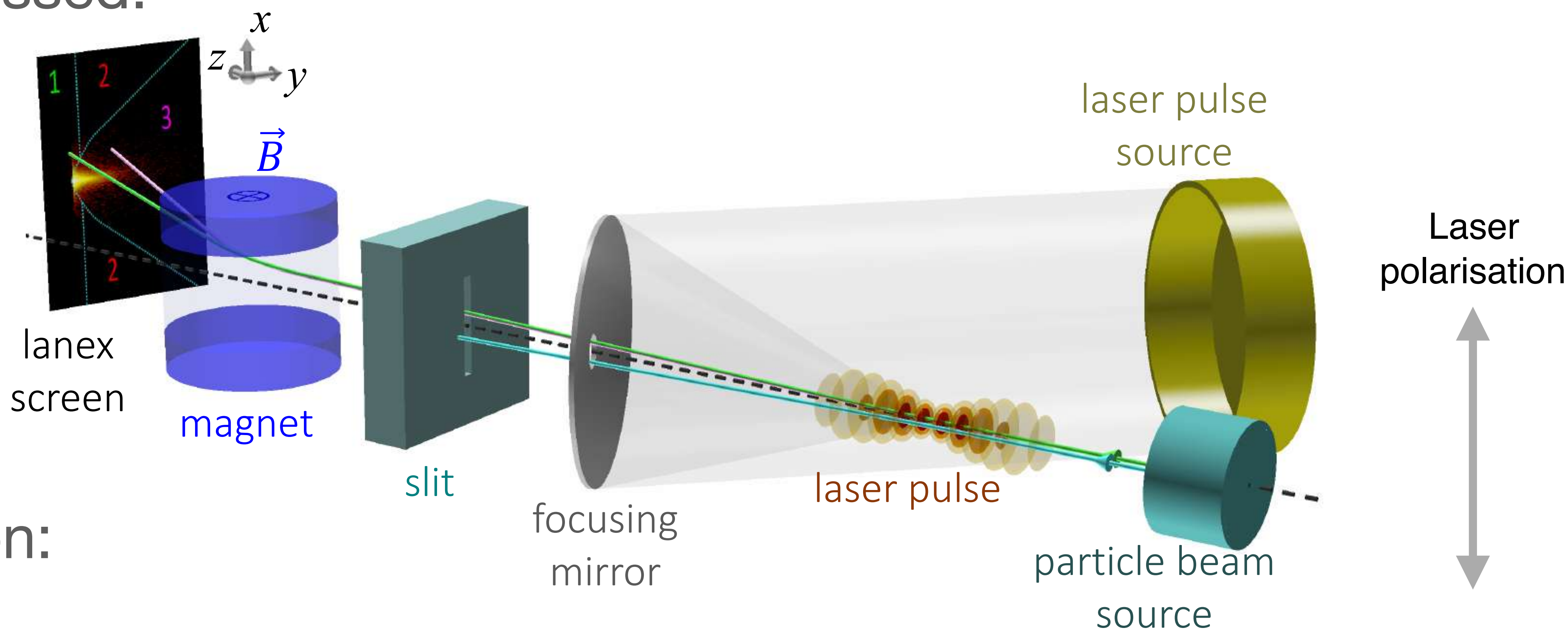


Top panel shows electron (420 MeV) trajectories in a collision with a laser pulse (a_0 = 200) for 1000 single electron QED simulations. Left panel is for a 1.4 fs pulse, right is for a 2.7 fs pulse.

Lower panel shows energy loss when the electrons pass through the pulse. Clear signal of quantum behavior in such an ideal case. Purple curves show shows classical radiation reaction.

Quantum quenching

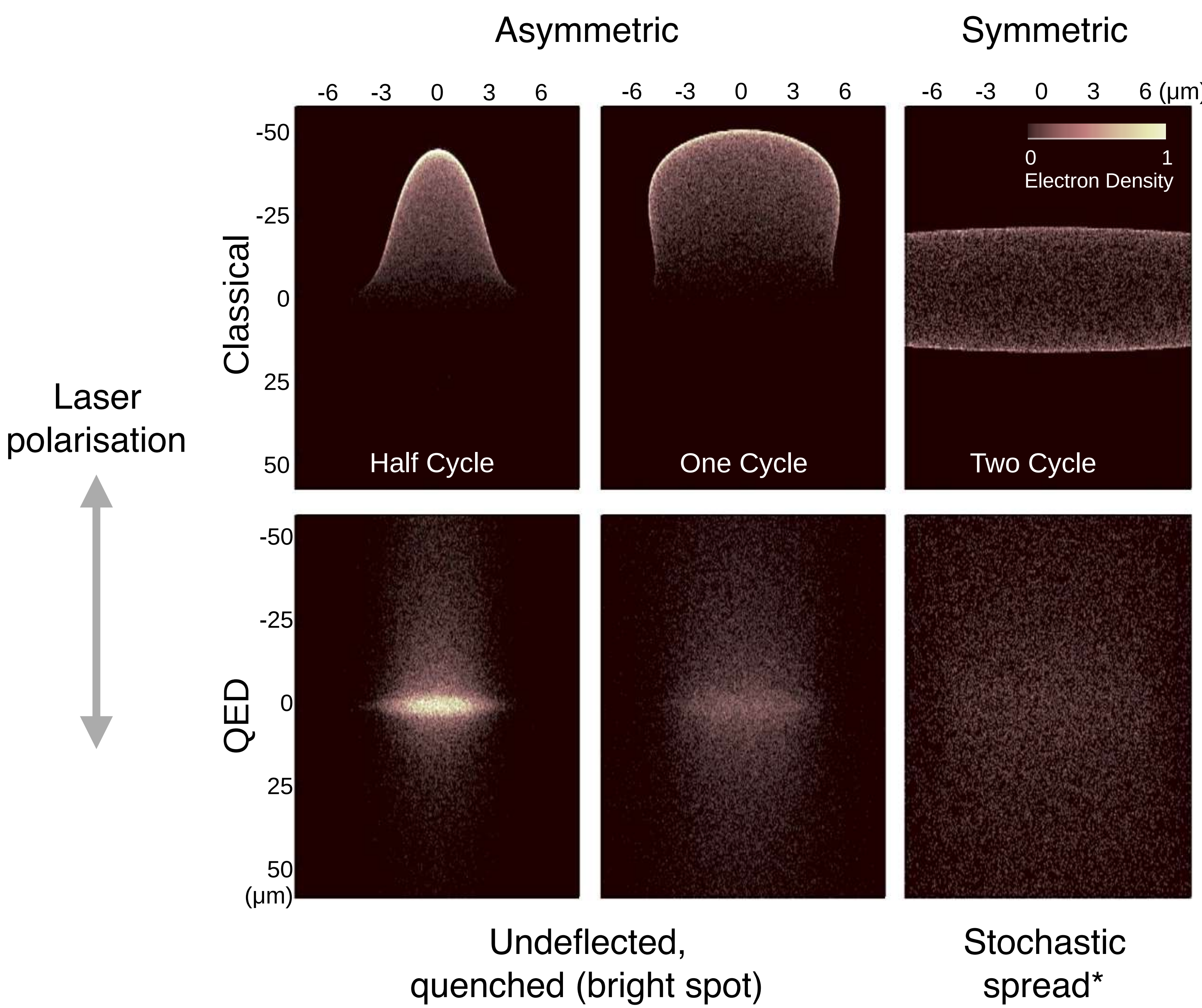
- Short pulse difficult. Effect also present for longer pulses, but suppressed.



- Solution:
  - Slit to sort out particles pushed by ponderomotive or tight focusing effects.
  - Magnet fans particles in *y*-direction according to energy.
  - Tight focusing: electrons in focus ⟶ larger *x*.

Final spatial electron distribution

- Towards ”realistic” electron bunch.

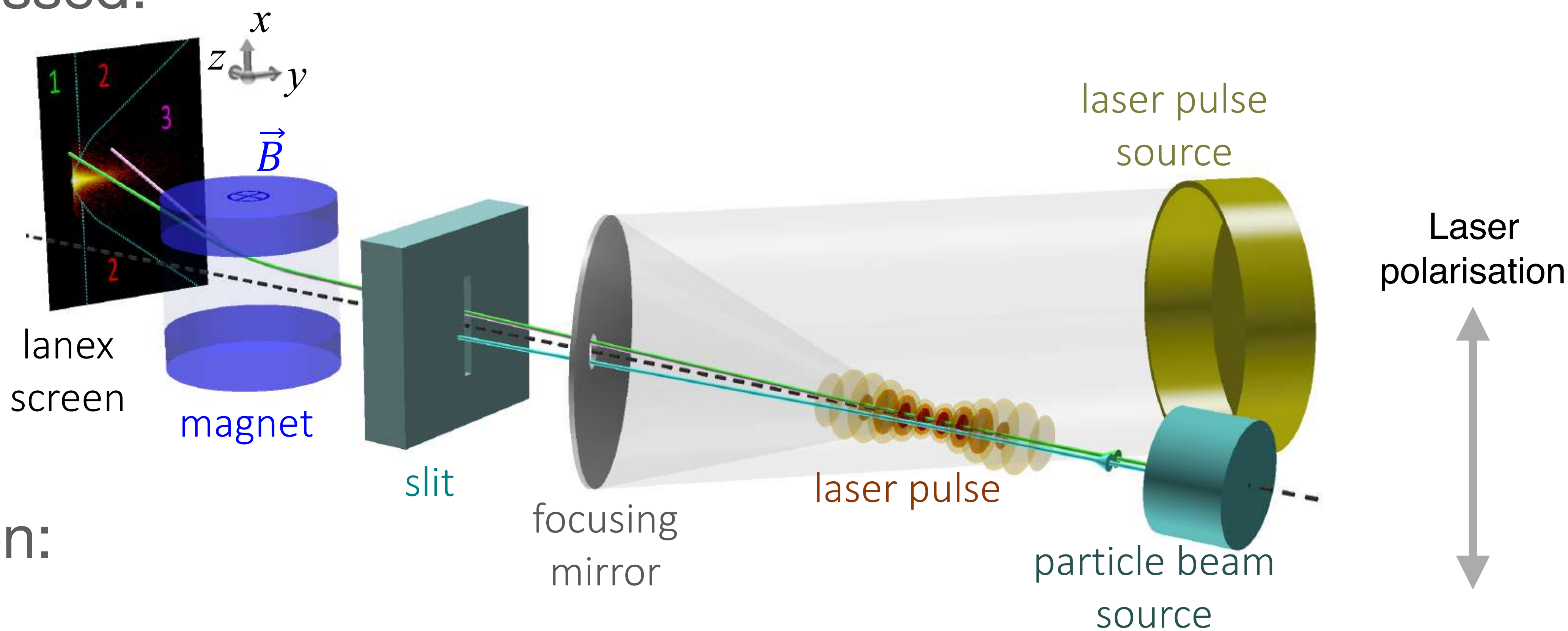


The electron bunch initially had Gaussian distributions in energy, 420 ± 0.35 MeV, and 2 μm FWHM. Laser intensity 200, focal spotsize 5 μm. One cycle: 2.7 fs.

*Green and Harvey, Phys. Rev. Lett. **112**, 164801 (2014).

Suggested experimental setup

- Short pulse difficult. Effect also present for longer pulses, but suppressed.



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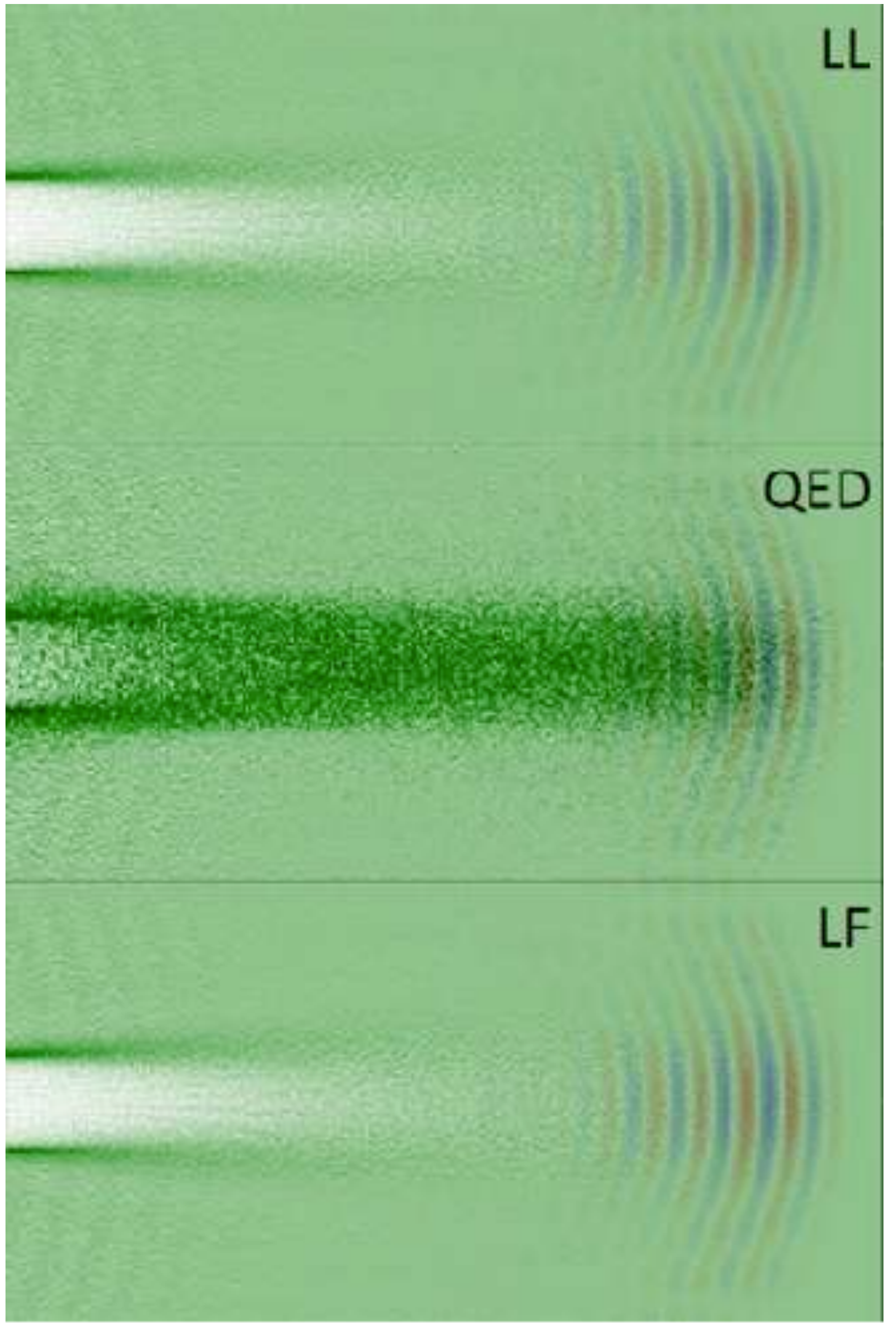
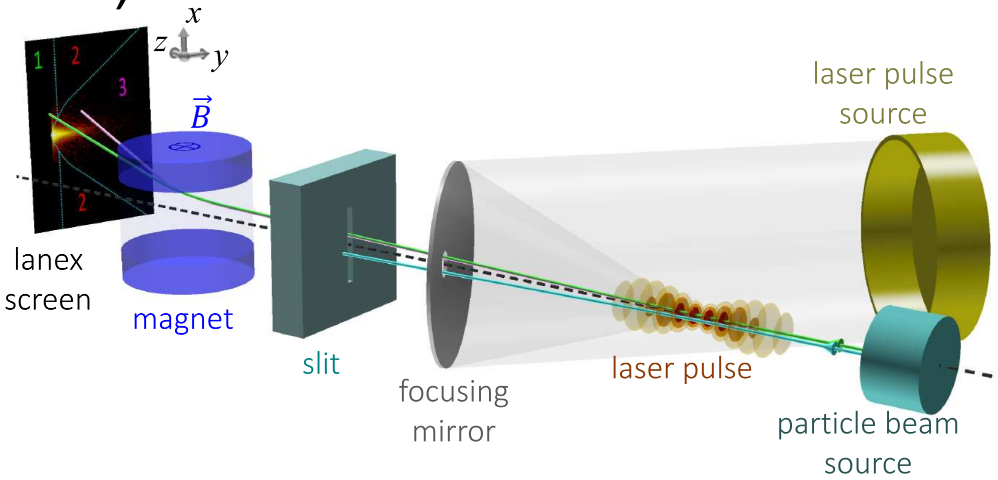
Simulated lanex screen

- Lanex screen simulations using 3D QED-PIC ELMIS3D.*



Laser: $a_0 = 50$ (50 TW, f/1), 820 nm.
Magnet: 0.32 T magnet, 1 m behind slit.
Electrons: 100 MeV (0.1% spread).
Lanex: 50 cm behind magnet, 45° angle.

- Close to centre: large deviation in x-direction, due to right focusing.
- Large energy loss: spread in *y*-direction.
- Small energy loss: left side of lanex, spread in *x*-direction.
- Slit removes spurious electrons in *y*-direction (e.g., by ponderomotive spread).



*Gonoskov et al., Phys. Rev. E **92**, 023305 (2015)

Simulated lanex screen

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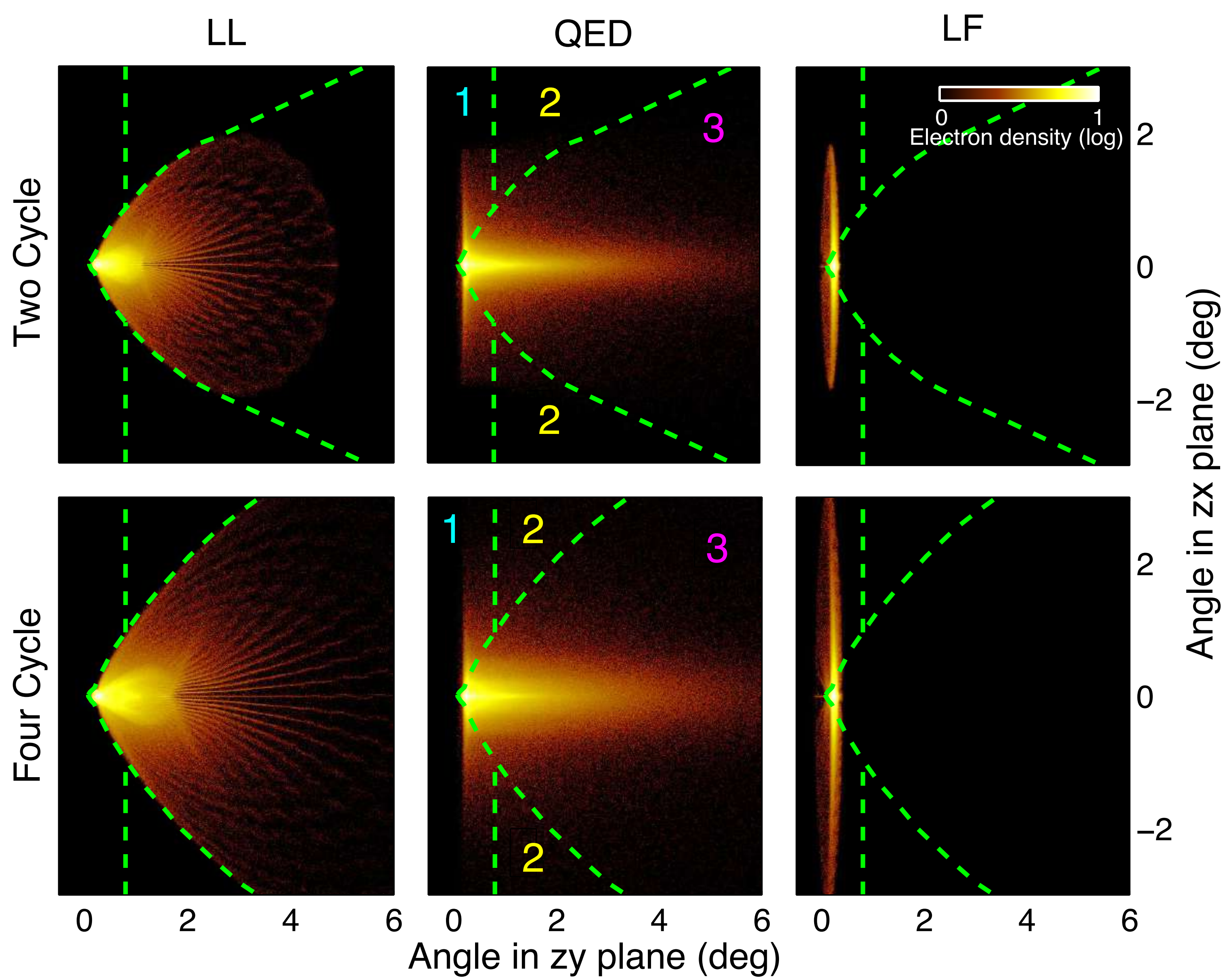

*Gonoskov et al., Phys. Rev. E **92**, 023305 (2015)

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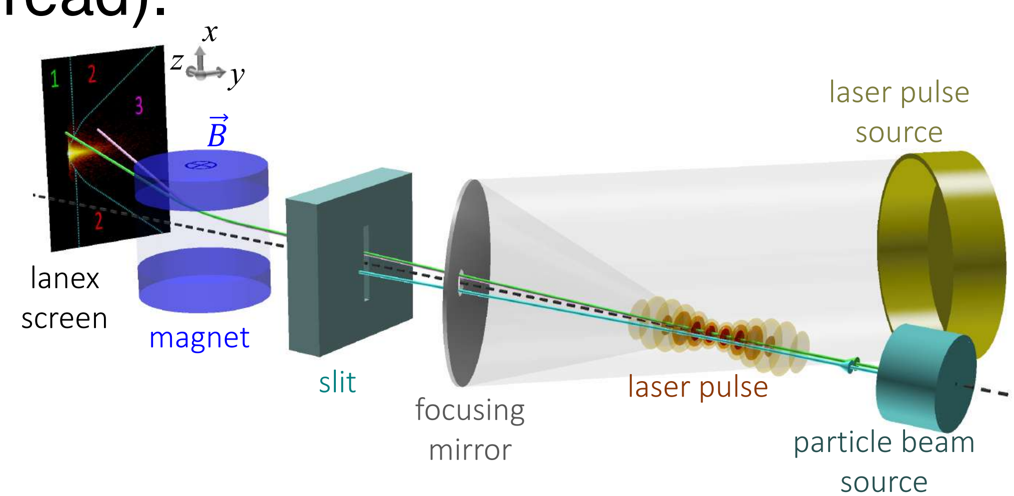
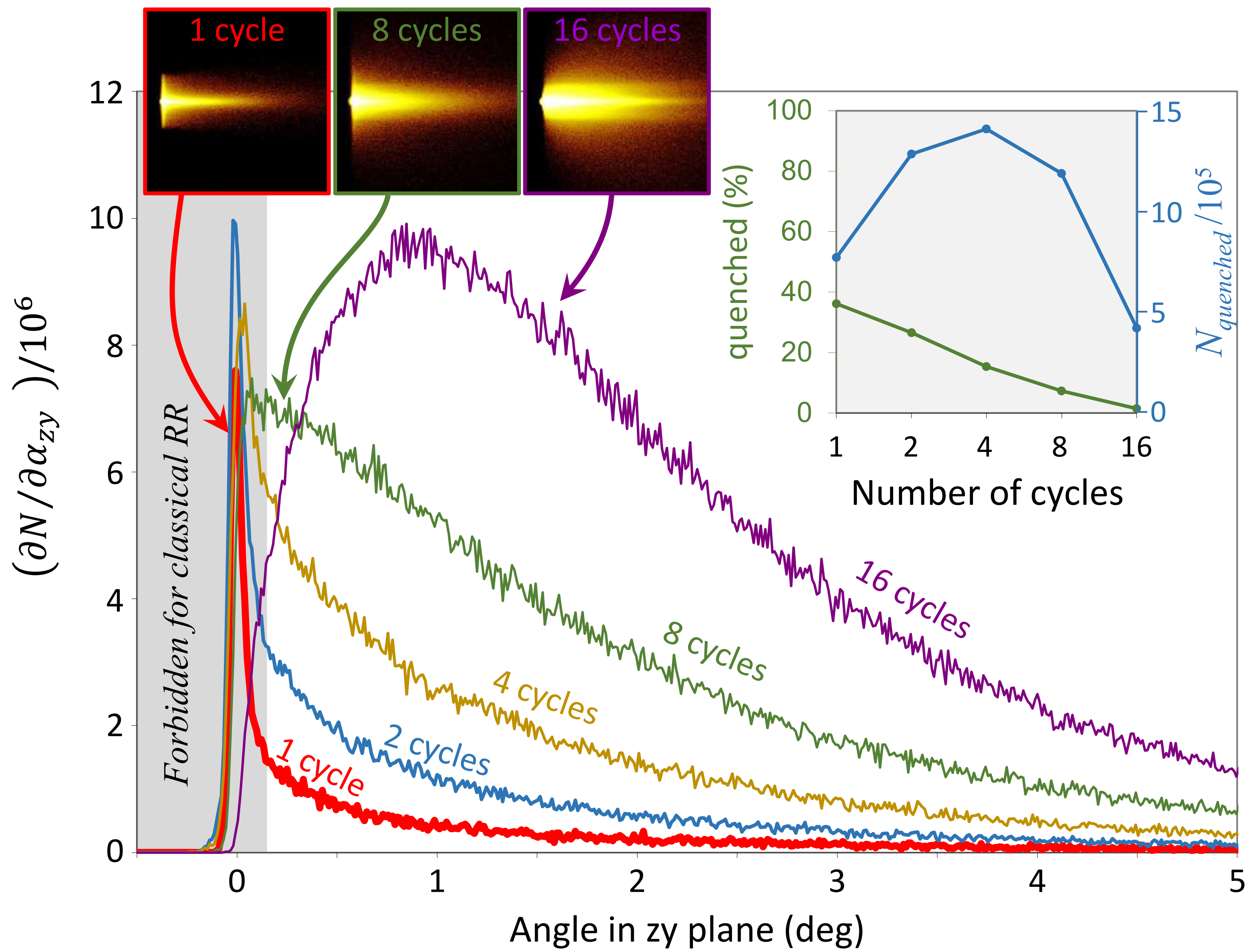


Pulse length

- Because of slit and magnet fanning, can relax pulse conditions.
- Even with 16 cycles, we obtain approx. 2% quenching.

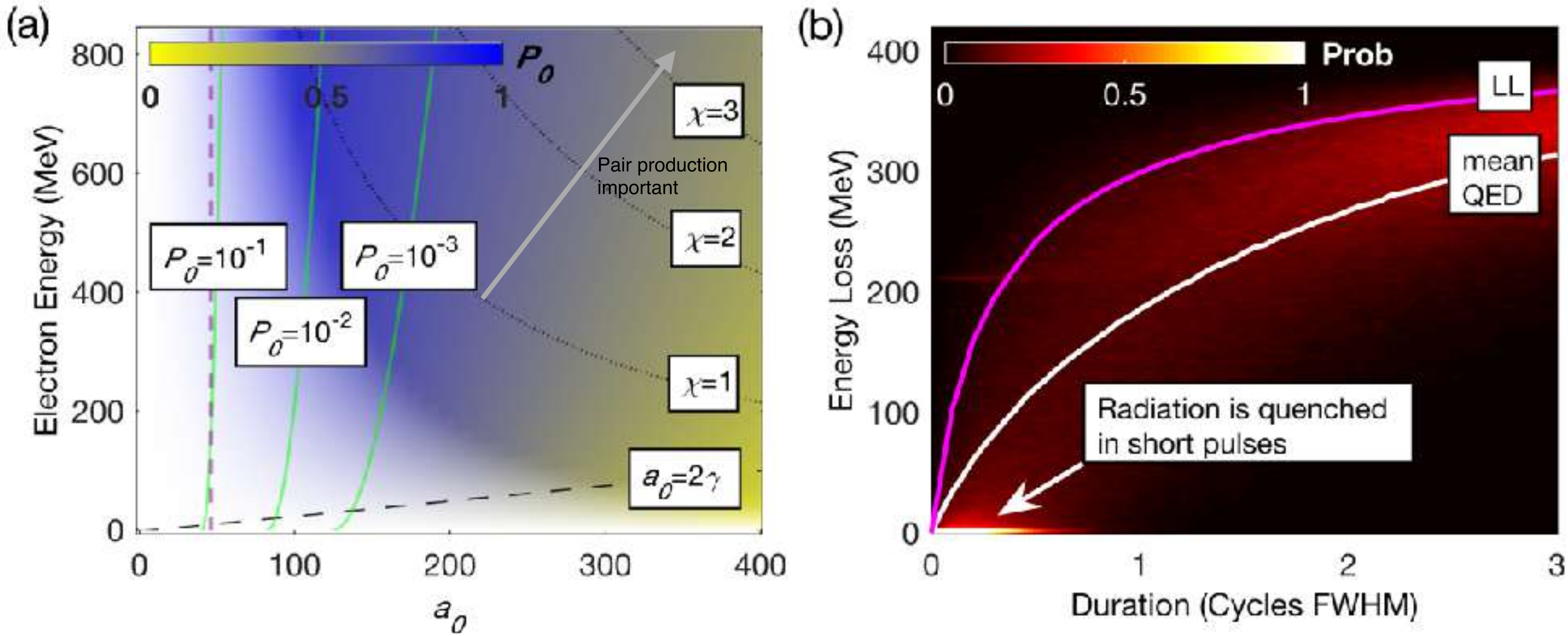


Conclusions

- Interesting to test fundamental aspects of photon emission and electron motion.
- No classic analogue.
- Surprising (?) possibility to switch radiation reaction off in the quantum regime.
- Deeper understanding of electron dynamics in strong fields.
- Energy loss depends on carrier-envelope phase; possibility to probe properties of laser pulse?
- Tight focusing: pair production setup even for 50 TW?

Probability distribution for quantized emission

- Probability of zero emitted photons: $P_0 = \exp(-P_1)$



(a) One-cycle pulse probability as a function of a_0 and the electron energy. The laser has wavelength 820 nm and spot size 5 μm. Radiation dominated regime where significant recoil is expected (thus no prob for zero energy, zero intensity).

(b) Probability of energy loss. Electron energy is 420 MeV, a_0 = 200. High probability for zero energy loss for short pulses can be seen in the QED case.